QUIZ 14 SOLUTIONS: LESSONS 20-21
MARCH 9, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [3 pts] Compute f_{xy} where $f(x,y) = \ln(x^2+y)$.

$$f_x = \frac{\partial}{\partial x}\left(\ln(x^2+y)\right) = \frac{2x}{x^2+y}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{2x}{x^2+y}\right) = 2x\left[\frac{\partial}{\partial y}\left(\frac{1}{x^2+y}\right)\right]$$

$$= 2x\left[\frac{\partial}{\partial y}(x^2+y)^{-1}\right]$$

$$= 2x\left[-1(x^2+y)^{-2}\right]$$

$$= \boxed{\frac{-2x}{(x^2+y)^2}}$$

$\boxed{\text{OR}}$

$$f_y = \frac{\partial}{\partial y}\left(\ln(x^2+y)\right) = \frac{1}{x^2+y}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{1}{x^2+y}\right)$$

$$= \frac{\partial}{\partial x}(x^2+y)^{-1}$$

$$= -1(2x)(x^2+y)^{-2}$$

$$= \boxed{\frac{-2x}{(x^2+y)^2}}$$

2. [7 pts] Suppose a book supplier estimates it prints

$$P(x,y) = 15x^{2/3}y^{1/3} \text{ thousand books}$$

where x is the number of employees and y is the amount of money invested in thousands of dollars.

Approximate the change in the number of books produced if the number of employees is increased from **75** to **100** and the amount of money invested is decreased from **\$20,000** to **\$18,000**. Round your answer to 3 decimal places.

$$\Delta P \approx \frac{\partial P}{\partial x}\Delta x + \frac{\partial P}{\partial y}\Delta y$$

$x = 75 \qquad \Delta x = +25$

$y = 20 \qquad \Delta y = -2$

$P(x,y) = 15x^{2/3}y^{1/3}$

y is measured in thousands

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(15x^{2/3}y^{1/3}\right) = 15y^{1/3}\frac{\partial}{\partial x}\left(x^{2/3}\right) = 15\left(\frac{2}{3}\right)y^{1/3}x^{-1/3}$$

$$= 10y^{1/3}/x^{1/3}$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\left(15x^{2/3}y^{1/3}\right) = 15x^{2/3}\frac{\partial}{\partial y}\left(y^{1/3}\right) = 15\left(\frac{1}{3}\right)x^{2/3}y^{-2/3}$$

$$= 5x^{2/3}/y^{2/3}$$

$x = 75$ $\quad \Delta x = 25$

$y = 20$ $\quad \Delta y = -2$

$$\Delta P \approx \frac{10(20)^{1/3}}{(75)^{1/3}}(25) + \frac{5(75)^{2/3}}{(20)^{2/3}}(-2)$$

$$= \boxed{136.778}$$