Chain Rule: $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

$z = f(x,y)$

$x = x(t)$, $y = y(t)$

QUIZ 15: LESSON 22
MARCH 19, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Evaluate $\frac{dz}{dt}$ at $t = 1$ if

$$z = e^{x^2+4xy+y^2+3y},\ x = \cos\left(\frac{\pi}{2}t\right),\text{ and } y = \ln t.$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(e^{x^2+4xy+y^2+3y}\right) \overset{\text{chain rule}}{=} \left[\frac{\partial}{\partial x}(x^2+4xy+y^2+3y)\right]e^{x^2+4xy+y^2+3y}$$

$$= (2x+4y)e^{x^2+4xy+y^2+3y}$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(e^{x^2+4xy+y^2+3y}\right) \overset{\text{Chain Rule}}{=} \left[\frac{\partial}{\partial y}(x^2+4xy+y^2+3y)\right]e^{x^2+4xy+y^2+3y}$$

$$= (4x+2y+3)e^{x^2+4xy+y^2+3y}$$

$$\frac{dx}{dt} = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right),\quad \frac{dy}{dt} = \frac{1}{t}$$

$$x(1) = \cos\left(\frac{\pi}{2}\right) = 0,\quad y(1) = \ln 1 = 0$$

$$\frac{dz}{dt}(t=1) = \underbrace{(2(0)+4(0))e^0}_{\frac{\partial z}{\partial x}(t=1)}\underbrace{\left(-\frac{\pi}{2}\right)}_{\frac{dx}{dt}(t=1)} + \underbrace{(4(0)+2(0)+3)e^0}_{\frac{\partial z}{\partial y}(t=1)}\underbrace{\left(\frac{1}{1}\right)}_{\frac{dy}{dt}(t=1)}$$

$$= \boxed{3}$$

2. [5 pts] The surface area of a cylinder is given by

$$SA(h,r) = 2\pi r^2 + 2\pi r h$$

where h is the height of the cylinder and r is the radius. Suppose

- the height of the cylinder is *decreasing* at a rate of **4** inches per minute
- the radius of the cylinder is *increasing* at a rate of **2** inches per minute

What is the rate of change of the surface area when the height is **10** inches and the radius is **15** inches?

Given: $\frac{dh}{dt} = -4, \quad \frac{dr}{dt} = +2$ Goal: $\frac{dSA}{dt}(h=10, r=15)$

$$\frac{\partial SA}{\partial h} = \frac{\partial}{\partial h}\left(2\pi r^2 + 2\pi r h\right) = 2\pi r$$

$$\frac{\partial SA}{\partial r} = \frac{\partial}{\partial r}\left(2\pi r^2 + 2\pi r h\right) = 4\pi r + 2\pi h$$

$$\frac{\partial SA}{\partial h}(h=10, r=15) = 30\pi, \quad \frac{\partial SA}{\partial r}(h=10, r=15) = 80\pi$$

$$\frac{dSA}{dt} = \frac{\partial SA}{\partial h}\frac{dh}{dt} + \frac{\partial SA}{\partial r}\frac{dr}{dt}$$

$$= 30\pi(-4) + 80\pi(2)$$

$$= -120\pi + 160\pi$$

$$= +40\pi$$

The surface area is <u>increasing</u> at a rate of $\boxed{40\pi}$ inches per minute.