

QUIZ 16 SOLUTIONS: LESSON 23  
MARCH ~~21~~, 2018  
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Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the local minima and maxima of

$$f(x, y) = 8x^2 - xy + 6y^2 - 135x + 92y$$

if they exist.

$$f_x = 16x - y - 135 = 0 \Rightarrow y = 16x - 135$$

$$f_y = -x + 12y + 92 = 0$$

substituting y  
 $\Rightarrow$

$$-x + 12(16x - 135) + 92 = 0$$

$$-x + 192x - 1620 + 92 = 0$$

$$191x - 1528 = 0$$

$$\Rightarrow x = \frac{1528}{191} = 8$$

$$y = 16(8) - 135 = -7$$

Critical pt:  $(8, -7)$

$$f_{xx} = 16, f_{yy} = 12, f_{xy} = -1 \Rightarrow D = 16(12) - (-1)^2 = 191 > 0$$

Because  $f_{xx} > 0$ ,  $(8, -7)$  is a local min pt. There is no local maxima.

$$\text{local min: } f(8, -7) = 8(8)^2 - 8(-7) + 6(-7)^2 - 135(8) + 92(-7) \\ = \boxed{-862}$$

2. [5 pts] Find (but do not classify) the critical points of

$$g(u, v) = -6u^2v + 48uv - 2v^2.$$

$$g_u = -12uv + 48v = -12v(u - 4)$$

$$g_v = -6u^2 + 48u - 4v$$

$$g_u = 0 \Rightarrow \text{either } v = 0 \text{ or } u = 4$$

Case 1: Assume  $v = 0$

$$0 = g_v = -6u^2 + 48u - \underbrace{4v}_0 = -6u^2 + 48u = -6u(u - 8)$$

$$\Rightarrow u = 0 \text{ or } u = 8$$

Critical pts:  $(0, 0), (8, 0)$

Case 2: Assume  $u = 4$

$$0 = g_v = -6\overset{\uparrow}{u^2}_{16} + 48\overset{\uparrow}{u}_4 - 4v = -6(16) + 48(4) - 4v = 96 - 4v$$

$$\Rightarrow 0 = 96 - 4v \Rightarrow v = \frac{-96}{-4} = 24$$

Critical pts:  $(4, 24)$

All Critical pts:  $\boxed{(0, 0), (8, 0), (4, 24)}$