Method of Lagrange Multipliers

$f_x = \lambda g_x$

$f_y = \lambda g_y$

$g(x,y) = C$

Solve for (x,y)

QUIZ 17: LESSONS 25-26
MARCH 30, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the maximum value of

$$f(x,y) = x - 2y^2$$

subject to $x^2 + y^2 = 9$.

$f_x = 1 \quad g_x = 2x$

$f_y = -4y \quad g_y = 2y$

$\Rightarrow \quad 1 = 2\lambda x$

$-4y = 2\lambda y$

$x^2+y^2=9$

$1 = 2\lambda x \overset{\lambda\neq 0,\, x\neq 0}{\Longrightarrow} 2\lambda = \frac{1}{x}$

$-4y = 2\lambda y \Rightarrow -4y = \frac{1}{x}\cdot y \Rightarrow -4xy = y \Rightarrow 0 = y + 4xy$

$= y(1+4x)$

Case 1: $y = 0$

$x^2 + 0^2 = 9 \Rightarrow x = \pm 3$

$(3,0),\ (-3,0)$

Case 2: $1+4x = 0 \Rightarrow x = -\frac{1}{4}$

$\left(-\frac{1}{4}\right)^2 + y^2 = 9 \Rightarrow \frac{1}{16} + y^2 = 9 \Rightarrow y^2 = 9 - \frac{1}{16}$

$\Rightarrow y = \pm\sqrt{9-\frac{1}{16}}$

$\left(-\frac{1}{4}, \sqrt{9-\frac{1}{16}}\right), \left(-\frac{1}{4}, -\sqrt{9-\frac{1}{16}}\right)$

$(3,0): \quad 3 - 2(0)^2 = 3$

$(-3,0): \quad -3 - 2(0)^2 = -3$

$\left(-\frac{1}{4}, \pm\sqrt{9-\frac{1}{16}}\right) = -\frac{1}{4} - 2\left(\pm\sqrt{9-\frac{1}{16}}\right)^2$

$= -\frac{1}{4} - 2\left(9 - \frac{1}{16}\right) < 0$

Max value: $\boxed{3}$

2. [5 pts] Suppose an artist sell sketches of her cat and dog online. She can make a profit of

$$P(x,y) = x^{3/2}y^{1/2} \text{ dollars/day}$$

if she offers x sketches of her cat and y sketches of her dog each day. If she is only able to create 32 sketches a day, what is the maximum profit she can make per day? Round your answer to the nearest cent.

$f(x,y) = x^{3/2}y^{1/2}, \quad g(x,y) = x+y = 32$

$f_x = \frac{3}{2}x^{1/2}y^{1/2} \qquad g_x = 1$

$f_y = \frac{1}{2}x^{3/2}y^{-1/2} \qquad g_y = 1$

$\Rightarrow \quad \frac{3}{2}x^{1/2}y^{1/2} = \lambda, \quad \frac{1}{2}x^{3/2}y^{-1/2} = \lambda, \quad x + y = 32$

$\Rightarrow \frac{3}{2}x^{1/2}y^{1/2} = \frac{1}{2}x^{3/2}y^{-1/2} \Rightarrow \frac{3}{2}x^{1/2}y^{1/2} - \frac{1}{2}x^{3/2}y^{-1/2} = 0$

$\Rightarrow \frac{1}{2}x^{1/2}y^{-1/2}(3y - x) = 0$

NOTE: $y \neq 0$ else f_y does not exist

Case 1: $x = 0$ $\quad (0,32)$

$0 + y = 32 \Rightarrow y = 32$

Case 2: $3y - x = 0 \Rightarrow x = 3y$ $\quad (24,8)$

$x + y = 32 = 3y + y = 32 \Rightarrow 4y = 32 \Rightarrow y = 8$

$x = 3(8) = 24$

$P(0,32) = 0, \quad P(24,8) = (24)^{3/2}(8)^{1/2} \approx \boxed{332.55}$

Max