QUIZ 19 SOLUTIONS: LESSONS 29
APRIL 6, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Compute

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

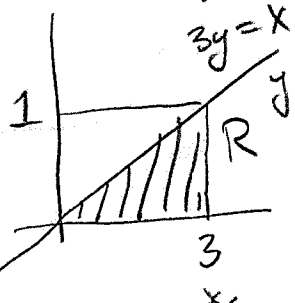
$0 \le y \le 1$

$3y \le x \le 3$

We need to swap the order of integration because $\int e^{x^2} dx$ cannot be written as an "elementary function". We write our region with x the independent variable:

$3y = x$

$\Rightarrow y = \frac{x}{3}$



$0 \le x \le 3$

$0 \le y \le \frac{x}{3}$

$$\int_0^1 \int_{3y}^3 e^{x^2} dx\, dy = \int_0^3 \int_0^{x/3} e^{x^2} dy\, dx = \int_0^3 y e^{x^2} \Big|_{y=0}^{y=\frac{x}{3}} dx$$

$$= \int_0^3 \frac{x e^{x^2}}{3} dx \quad \leftarrow \text{u-sub: } u = x^2,\ du = 2x\,dx,\ u(3) = 3^2 = 9,\ u(0) = 0^2 = 0$$

$$= \int_0^9 \frac{1}{6} e^u du$$

$$= \frac{1}{6} e^u \Big|_0^9 = \frac{1}{6} e^9 - \frac{1}{6} e^0 = \boxed{\frac{1}{6}(e^9 - 1)}$$

2. [5 pts] Compute

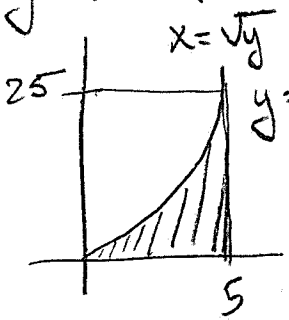
$$\int_0^{25}\int_{\sqrt{y}}^{5}\sqrt{x^3+1}\,dx\,dy.$$

$0 \le y \le 25$
$\sqrt{y} \le x \le 5$

Round your answer to 3 decimal places.

We do not have the tools to integrate $\sqrt{x^3+1}$ with respect to x, so we swap the order of integration to see if we have better luck.

$\sqrt{y} = x \Leftrightarrow y = x^2$



$\longrightarrow$ $0 \le x \le 5$, $0 \le y \le x^2$

$$\int_0^{25}\int_{\sqrt{y}}^{5}\sqrt{x^3+1}\,dx\,dy = \int_0^5\int_0^{x^2}\sqrt{x^3+1}\,dy\,dx$$

$$= \int_0^5 y\sqrt{x^3+1}\Big|_{y=0}^{y=x^2}\,dx$$

$$= \int_0^5 x^2\sqrt{x^3+1}\,dx$$

u-sub: $u = x^3+1$, $du = 3x^2\,dx$, $u(5) = 5^3+1 = 126$, $u(0) = 0^3+1 = 1$

$$= \int_1^{126}\frac{1}{3}u^{1/2}\,du$$

$$= \frac{1}{3}\left(\frac{2}{3}\right)u^{3/2}\Big|_1^{126}$$

$$= \frac{2}{9}(126)^{3/2} - \frac{2}{9}$$

$$\approx \boxed{814.077}$$