

QUIZ 20: LESSON 30
APRIL 9, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Solve the system of equations

$$\begin{cases} x + 2y + z = 2 \\ -x - y - z = -2 \\ 2x + y + z = 1 \end{cases}$$

Translate \rightarrow

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ -1 & -1 & -1 & -2 \\ 2 & 1 & 1 & 1 \end{array} \right] \xrightarrow{\substack{R_1+R_2 \\ \rightarrow R_2}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 \end{array} \right] \xrightarrow{\substack{-2R_1+R_3 \\ \rightarrow R_3}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & -3 \end{array} \right]$$

$$\xrightarrow{\substack{3R_2+R_3 \\ \rightarrow R_3}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -3 \end{array} \right] \xrightarrow{-R_3 \rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{\substack{-R_3+R_1 \\ \rightarrow R_1}} \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\xrightarrow{\substack{-2R_2+R_1 \\ \rightarrow R_1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{\text{Translate}} \begin{cases} x = -1 \\ y = 0 \\ z = 3 \end{cases}$$

$$(x, y, z) = \boxed{(-1, 0, 3)}$$

2. [5 pts] Suppose a goldsmith has two alloys of gold:

- Alloy A has a gold purity of 72%

- Alloy B has a gold purity of 86%

The goldsmith mixes x grams of Alloy A with y grams of Alloy B such that he obtains 50 grams of Alloy C, which has a gold purity of 80%. Find x to the nearest gram.

$x =$ grams of Alloy A, 72% of A is gold
 $y =$ grams of Alloy B, 86% of B is gold

System of Equations:

$$\begin{cases} x + y = 50 & \leftarrow \text{Total grams of A + B = amount of C} \\ .72x + .86y = .8(50) & \leftarrow \text{Amount of gold contributed by A and B must} \\ & \text{make 80\% of C (=50 grams) gold} \end{cases}$$

simplify \rightarrow

$$\begin{cases} x + y = 50 \\ 72x + 86y = 80(50) \end{cases} = \begin{cases} x + y = 50 \\ 72x + 86y = 4000 \end{cases}$$

Solution:

$$x + y = 50 \Rightarrow x = 50 - y$$

$$72x + 86y = 4000 \Rightarrow 72(50 - y) + 86y = 4000$$

$$\Rightarrow 3600 - 72y + 86y = 4000$$

$$\Rightarrow 14y = 400$$

$$\Rightarrow y = \frac{400}{14} = \frac{200}{7}$$

$$x = 50 - \frac{200}{7} = \frac{350}{7} - \frac{200}{7} = \frac{150}{7} \approx \boxed{21 \text{ g}}$$