QUIZ 21 SOLUTIONS: LESSON 31
APRIL 11, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Transform the following matrix into reduced row-echelon form:

$$\begin{cases} 2x & + & 3y & = & 1 \\ -x & + & 5y & = & -7 \end{cases}$$

Carefully label each row operation you use.

There are many ways to do this, what I do is just one possibility:

$$\begin{cases} 2x + 3y = 1 \\ -x + 5y = -7 \end{cases} \xrightarrow{\text{Translate}} \left[\begin{array}{cc|c} 2 & 3 & 1 \\ -1 & 5 & -7 \end{array}\right] \xrightarrow{R_2 + R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 8 & -6 \\ -1 & 5 & -7 \end{array}\right]$$

$$\xrightarrow{R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 8 & -6 \\ 0 & 13 & -13 \end{array}\right] \xrightarrow{R_2/13 \to R_2} \left[\begin{array}{cc|c} 1 & 8 & -6 \\ 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{-8R_2 + R_1 \to R_1} \boxed{\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]}$$

2. [5 pts] Solve the following system of equations:

$$\begin{cases} -x + y - z = -1 \\ 2x - y - z = 4 \\ 3x + 2y + 2z = -1 \end{cases}$$

Again, there are many ways to do this.

$$\xrightarrow{\text{Translate}} \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1 \\ 2 & -1 & -1 & 4 \\ 3 & 2 & 2 & -1 \end{array}\right] \xrightarrow{R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 2 & -1 & -1 & 4 \\ 3 & 2 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{-2R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & -1 & 3 & -2 \\ 3 & 2 & 2 & -1 \end{array}\right] \xrightarrow{-3R_1+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & -1 & 3 & -2 \\ 0 & 2 & 8 & -10 \end{array}\right]$$

$$\xrightarrow{R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 11 & -12 \\ 0 & 2 & 8 & -10 \end{array}\right] \xrightarrow{-2R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 11 & -12 \\ 0 & 0 & -14 & 14 \end{array}\right]$$

$$\xrightarrow{R_3/-14\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 11 & -12 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow{-11R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{2R_3+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\boxed{(x,y,z) = (1,-1,-1)}$$