QUIZ 23 SOLUTIONS: LESSON 33
APRIL 18, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Use row operations to find the inverse of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Carefully label each row operation you use.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \xrightarrow[\to R_2]{-3R_1+R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right]$$

$$\xrightarrow[\to R_1]{R_2+R_1} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right] \xrightarrow{-\frac{R_2}{2} \to R_2} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$

Check:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -2+3 & 1-1 \\ -6+6 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2+3 & -4+4 \\ \frac{3}{2}-\frac{3}{2} & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

2. [5 pts] The inverse matrix of

$$B = \begin{bmatrix} -3/4 & 5/4 & -3/2 \\ -1 & 1 & -1 \\ -1/4 & 3/4 & -1/2 \end{bmatrix}$$

is

$$B^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 0 & 3 \\ -2 & 1 & 2 \end{bmatrix}$$

Given the related system of equations

$$\begin{aligned} -\frac{3}{4}x + \frac{5}{4}y - \frac{3}{2}z &= 3 \\ -x + y - z &= -1 \\ -\frac{1}{4}x + \frac{3}{4}y - \frac{1}{2}z &= 2 \end{aligned},$$

find the solution (x, y, z).

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $Y = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$. Then $BX = Y \Rightarrow X = B^{-1}Y$.

So

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 0 & 3 \\ -2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3+2+2 \\ -3+0+6 \\ -6-1+4 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ -3 \end{bmatrix}.$$

$$(x, y, z) = \boxed{(7, 3, -3)}$$