Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [3 pts] Evaluate $\int \dfrac{3x}{\sqrt{x-4}}\, dx$.

$$\sqrt{u} = u^{1/2}, \quad \frac{1}{\sqrt{u}} = u^{-1/2}$$

$u = x - 4$

$\Rightarrow x = u + 4$

$du = dx$

$$\int \frac{3x}{\sqrt{x-4}}\, dx = \int \frac{3(u+4)}{\sqrt{u}}\, du = \int \left(\frac{3u}{\sqrt{u}} + \frac{12}{\sqrt{u}}\right) du$$

$$= \int \left(3u^{1/2} + 12u^{-1/2}\right) du$$

$$= 3\left(\frac{1}{\frac{1}{2}+1} u^{\frac{1}{2}+1}\right) + 12\left(\frac{1}{-\frac{1}{2}+1} u^{-\frac{1}{2}+1}\right) + C$$

$$= 3\left(\frac{2}{3}\right)u^{3/2} + 12(2)u^{1/2} + C = \boxed{2(x-4)^{3/2} + 24(x-4)^{1/2} + C}$$

Note: Once you integrate, drop the integral symbol

2. [3 pts] Evaluate $\int_1^3 7(x^2-3)(x^3-9x)^5\, dx$. Round your answer to 3 decimal places.

$u = x^3 - 9x$

$du = (3x^2 - 9)dx \Rightarrow \dfrac{du}{3} = (x^2-3)dx$

$\quad = 3(x^2-3)dx$

$u(3) = (3)^3 - 9(3) = 0$

$u(1) = (1)^3 - 9(1) = -8$

$$\int_1^3 7(x^2-3)(x^3-9x)^5\, dx$$

$$= 7\int_1^3 (x^3-9x)^5 (x^2-3)\, dx$$
$$\underbrace{\quad}_{u} \qquad \underbrace{\quad}_{du/3}$$

$$= 7 \int_{u(1)=-8}^{u(3)=0} u^5\, \frac{du}{3}$$

$$= \frac{7}{3} \int_{-8}^{0} u^5\, du$$

$$= \frac{7}{3}\left(\frac{1}{5+1}\right)u^{5+1}\Big|_{-8}^{0} = \frac{7}{3}\left(\frac{1}{6}(0)^6 - \frac{1}{6}(-8)^6\right)$$

$$= \frac{-7}{18}(-8)^6 \approx \boxed{-101{,}944.88}$$

1

3. [4 pts] Find the average of the function $7x^2 e^{x^3} + 2$ over the interval $0 \le x \le 4$.

Average Value formula: $\frac{1}{b-a} \int_a^b f(x)\,dx$

$a = 0, b = 4$

Average: $\frac{1}{4-0} \int_0^4 (7x^2 e^{x^3} + 2)\,dx = \frac{1}{4} \int_0^4 (7x^2 e^{x^3} + 2)\,dx$

$u = x^3$

$du = 3x^2\,dx \Rightarrow \frac{du}{3} = x^2\,dx$

$u(4) = 4^3 = 64$

$u(0) = 0^3 = 0$

$= \frac{7}{4} \int_0^4 x^2 e^{x^3}\,dx + \frac{1}{4} \int_0^4 2\,dx$

$= \frac{7}{4} \int_{u(0)=0}^{u(4)=64} e^u \left(\frac{du}{3}\right) + \frac{1}{4} \int_0^4 2\,dx$

The u only applies to this integral $\longrightarrow$

$= \frac{7}{12} \int_0^{64} e^u\,du + \frac{1}{4} \int_0^4 2\,dx$

$= \frac{7}{12} \left(e^u\right)\Big|_0^{64} + \frac{1}{4}(2x)\Big|_0^4$

$= \frac{7}{12}\left(e^{64} - \underbrace{e^0}_{1}\right) + \underbrace{\frac{1}{2}(4-0)}_{2}$

$= \frac{7}{12}e^{64} - \frac{7}{12} + 2$

$= \frac{7}{12}e^{64} - \frac{7}{12} + \frac{24}{12}$

$= \boxed{\frac{7}{12}e^{64} + \frac{17}{12}}$

This problem is more complicated than what was presented in the practice quiz questions. Full points will be awarded to the correct setup and choice of u.