QUIZ 5 SOLUTIONS: LESSONS 5-6
JANUARY 26, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [6 pts] Suppose a car has a velocity of $55te^{-t/55}$ miles per hour where t is time in hours. How far has the car traveled in an hour? Round your answer to 2 decimal places.

The question asks us to compute $\int_0^1 55te^{-t/55}\,dt$. This is an integration by parts problem. By LIATE,

$u = t$ $\quad dv = 55e^{-t/55}\,dt$

$du = dt$ $\quad v = \int 55e^{-t/55}\,dt = \int 55e^{w}(-55\,dw)$

$w = -t/55$

$dw = -\frac{1}{55}dt$ $\quad = -55^2\int e^w\,dw$

$-55\,dw = dt$ $\quad = -55^2 e^{-t/55}$

$$\int_0^1 55te^{-t/55}\,dt = \underbrace{t}_{u}\underbrace{\left(-55^2e^{-t/55}\right)}_{v}\Big|_0^1 - \int_0^1 \underbrace{\left(-55^2e^{-t/55}\right)}_{v}\underbrace{(dt)}_{du}$$

$$= -55^2te^{-t/55}\Big|_0^1 + 55^2\int_0^1 e^{-t/55}\,dt$$

$$= -55^2te^{-t/55}\Big|_0^1 - 55^3e^{-t/55}\Big|_0^1$$

(By our work for v)

$$= -55^2te^{-t/55} - 55^3e^{-t/55}\Big|_0^1$$

$$= -55^2(1)e^{-1/55} - 55^3e^{-1/55} - \left(-55^2(0)\underbrace{e^0}_{1} - 55^3\underbrace{e^0}_{1}\right)$$

$$= -55^2e^{-1/55} - 55^3e^{-1/55} + 55^3$$

$$\approx \boxed{27.17}$$

2. [4 pts] Solve for y as a function of t given $y' = -20\dfrac{t^2}{y}$.

$$\frac{dy}{dt} = y' = -20\frac{t^2}{y}$$

$$y\frac{dy}{dt} = -20t^2$$

$$y\,dy = -20t^2\,dt$$

$$\int y\,dy = \int -20t^2\,dt$$

$$\frac{1}{2}y^2 = -\frac{20}{3}t^3 + C$$

$$y^2 = -\frac{40}{3}t^3 + C$$

$$\boxed{y = \sqrt{-\frac{40}{3}t^3 + C}}$$