QUIZ 6: LESSON 7
JANUARY 31, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Find the general solution to $\frac{dy}{dx} = x^2 e^{-x^3}$.

This is a separable equation

$$\frac{dy}{dx} = x^2 e^{-x^3}$$
$$\Rightarrow dy = x^2 e^{-x^3} dx$$
$$\Rightarrow \int dy = \int x^2 e^{-x^3} dx$$
$$\Rightarrow \boxed{y = -\frac{1}{3} e^{-x^3} + C}$$

$$\int x^2 e^{-x^3} dx = \int x^2 e^u \left(\frac{-du}{3x^2}\right)$$
$u = -x^3$
$du = -3x^2 dx$
$\frac{-du}{3x^2} = dx$
$$= \int -\frac{1}{3} e^u du$$
$$= -\frac{1}{3} e^u + C$$
$$= -\frac{1}{3} e^{-x^3} + C$$

2. [6 pts] Find the particular solution to $\frac{dy}{dt} + y \sin t = 0$ where $y(\pi) = 1$.

This is a separable equation, but we need to rewrite:

$$\frac{dy}{dt} + y \sin t = 0 \Rightarrow \frac{dy}{dt} = -y \sin t$$
$$\Rightarrow \frac{1}{y} dy = -\sin t \, dt$$
$$\Rightarrow \int \frac{1}{y} dy = \int -\sin t \, dt$$
$$\Rightarrow \ln|y| = \cos t + C$$

→

We apply e to both sides to get

$$e^{\ln|y|} = e^{\cos t + C} = e^{\cos t} e^{C} = Ce^{\cos t}$$

$$|y| = Ce^{\cos t}$$

We assume $y > 0$ and drop the absolute values. So the general solution is

$$y = Ce^{\cos t}$$

We were told $y(\pi) = 1$ so,

$$1 = Ce^{\cos \pi} = Ce^{-1}$$

$$\Rightarrow C = e$$

So

$$y = ee^{\cos t} = e^{1} e^{\cos t} = \boxed{e^{\cos t + 1}}$$