QUIZ 7: LESSONS 8 & 9
FEBRUARY 5, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] In a particular chemical reaction, Substance A is converted into Substance B at a rate proportional to the square of the amount of Substance A present at time t. If $y(t)$ is the amount of Substance A present at time t, write down a differential equation that describes this situation.

Substance A is changing at a rate proportional to the square of $y(t)$. So

$$\boxed{\frac{dy}{dt} = ky^2}$$

2. [6 pts] Find the general solution to $\dfrac{dy}{dx} - \dfrac{y}{x} = x^3$.

We apply the FOLDE technique. We also assume $x > 0$. (*)

Step 1: Find P, Q

$$P(x) = -\frac{1}{x}, \quad Q(x) = x^3$$

Step 2: Find integrating factor

$$u(x) = e^{\int P(x)\,dx} = e^{-\int \frac{1}{x}\,dx} \overset{(*)}{=} e^{-\ln(x)} = e^{\ln x^{-1}} = x^{-1}$$

Step 3: Solve

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The solution is given by

$$y \cdot u(x) = \int Q(x)\,u(x)\,dx$$

$$y \cdot \underbrace{x^{-1}}_{u(x)} = \int \underbrace{x^{3}}_{Q(x)} \cdot \underbrace{x^{-1}}_{u(x)}\,dx$$

$$y x^{-1} = \int x^{2}\,dx = \frac{1}{3}x^{3} + C$$

$$\boxed{y = \frac{1}{3}x^{4} + C \cdot x}$$