QUIZ 8 SOLUTIONS: LESSON 10
FEBRUARY 7, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

Setup: A 500-gallon tank initially contains 300 gallons of brine containing 20 pounds of dissolved salt. Brine containing 5 pounds of salt per gallon flows into the tank at a rate of 4 gallons per minute and the well-stirred mixture flows out of the tank at a rate of 3 gallons per minute.

1. [2 pts] Write down a differential equation describing the situation above where $A(t)$ is the number of pounds of salt in the tank at time t minutes.

$\frac{dA}{dt} = [\text{salt in}] - [\text{salt out}]$

$[\text{salt in}]$: $\left(\frac{5 \text{ lbs}}{1 \text{ gal}}\right)\left(\frac{4 \text{ gal}}{1 \text{ min}}\right) = 20 \text{ lbs/min}$

$[\text{salt out}]$: gal in tank at time t:

$300 + (4-3)t = 300 + t$

(4 in, 3 out)

$\left(\frac{A \text{ lbs}}{300+t \text{ gal}}\right)\left(\frac{3 \text{ gal}}{1 \text{ min}}\right) = \frac{3A}{300+t} \text{ lbs/min}$

$\frac{dA}{dt} = 20 - \frac{3A}{300+t}$ lbs/min

2. Assuming we are using the FOLDE method to solve # 1, answer the following:

(a) [1 pt] What are P and Q?

$\frac{dA}{dt} = 20 - \frac{3A}{300+t} \Rightarrow \frac{dA}{dt} + \frac{3A}{300+t} = 20$

$P(t) = \frac{3}{300+t}$, $Q(t) = 20$

(b) [2 pts] What is the integrating factor?

$u(t) = e^{\int P(t)\,dt}$

No +C for integrating factor

$\int P(t)\,dt = \int \frac{3}{300+t}\,dt = \int \frac{3}{u}\,du = 3\ln|u| = 3\ln|300+t|$

$u = 300+t$, $du = dt$

$300+t$ represents # of gal in tank, safely assume >0

$u(t) = e^{\int P(t)\,dt} = e^{3\ln(300+t)} = e^{\ln(300+t)^3} = (300+t)^3$

Recall: $a\ln b = \ln b^a$, $e^{a\ln b} = e^{\ln b^a} = b^a$

(c) [1 pt] What is $A(0)$?

We were told there are initially 20 lbs of salt in tank at time $t=0$, so $A(0) = 20$

(d) [2 pts] What is the particular solution to the differential equation in # 1?

$A\cdot u(t) = \int Q(t)\,u(t)\,dt$

$\Rightarrow A\cdot(300+t)^3 = \int 20(300+t)^3\,dt$

$A(300+t)^3 = 5(300+t)^4 + C$ (*)

$\Rightarrow A = 5(300+t) + \frac{C}{(300+t)^3}$

$20 = 5(300) + \frac{C}{(300)^3} \Rightarrow C = -1480(300)^3$

(*) $= \int 20(300+t)^3\,dt$

$w = 300+t$, $dw = dt$

$= \int 20w^3\,dw = \frac{20}{4}w^4 + C = 5w^4 + C = 5(300+t)^4 + C$

$A = 5(300+t) - \frac{1480(300)^3}{(300+t)^3}$

3. [2 pts] Find how many pounds of salt are in the tank when the tank is full. Round your answer to the nearest hundredth.

Tank is full when $\underbrace{300+t}_{\text{Total gal in tank at time } t} = 500$, so

$A(\text{time tank is full}) = 5(500) - \frac{1480(300)^3}{(500)^3} \approx 2180.32$ lbs