

QUIZ 9: LESSONS 11-12
FEBRUARY 12, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Find the area bounded by $y = x^2 - 3x$ and $y = -x + 3$.
 Round your answer to the nearest hundredth.

(1) Intersection points:

$$x^2 - 3x = -x + 3$$

$$x^2 - 3x + x - 3 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0 \Rightarrow x = -1, 3$$

(2) Which function is bigger on $[-1, 3]$?

Try $x=2$:

$$x^2 - 3x : 2^2 - 3(2) = 4 - 6 = -2$$

$$-x + 3 : -2 + 3 = 1 \leftarrow \text{Larger}$$

Area: $\int_{-1}^3 [(-x+3) - (x^2-3x)] dx = \int_{-1}^3 (-x^2 + 2x + 3) dx$

Bounds from (1) Larger Function Smaller Function

$$= -\frac{1}{3}x^3 + x^2 + 3x \Big|_{-1}^3$$

$$= -\frac{1}{3}(3)^3 + (3)^2 + 3(3) - \left[-\frac{1}{3}(-1)^3 + (-1)^2 + 3(-1) \right]$$

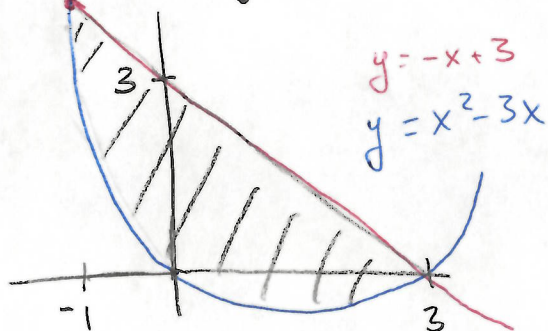
$$= -9 + 9 + 9 - \frac{1}{3} - 1 + 3$$

$$= 9 + 2 - \frac{1}{3}$$

$$= 11 - \frac{1}{3}$$

$$= \frac{33}{3} - \frac{1}{3} = \frac{32}{3} \approx \boxed{10.67}$$

Sketch of Region:



$$x^2 - 3x = 0$$

$$\Rightarrow x = 0, 3$$

2. [6 pts] Find the volume of the solid obtained by revolving the region enclosed by the curves

$$y = \sqrt{25 - \frac{x^2}{4}}, \quad y = 0, \quad x = 0$$

about the y -axis.

We are revolving about the y -axis which means we need a function of y . So we solve for x :

$$y = \sqrt{25 - \frac{x^2}{4}}$$

$$\Rightarrow y^2 = 25 - \frac{x^2}{4}$$

$$\Rightarrow \frac{x^2}{4} = 25 - y^2$$

$$\Rightarrow x^2 = 100 - 4y^2$$

$$\Rightarrow x = \sqrt{100 - 4y^2}$$

↑ Radius

We are integrating with respect to y , thus we need bounds in terms of y . Set $x=0$ to get

$$0 = \sqrt{100 - 4y^2} \Rightarrow 0 = 100 - 4y^2 \Rightarrow \frac{-100}{-4} = y^2$$

So $y = \pm 5$. But, our region is in the first quadrant, so our bounds are $0 \leq y \leq 5$

$$\text{Volume: } \int_0^5 \pi \left(\sqrt{100 - 4y^2} \right)^2 dy = \int_0^5 \pi (100 - 4y^2) dy$$

↑ Radius

$$= \pi \left[100y - \frac{4}{3}y^3 \right]_0^5$$

$$= \pi (100(5) - \frac{4}{3}(5)^3)$$

$$= \pi (500 - \frac{4}{3}(125))$$

$$= \pi \left(\frac{1500}{3} - \frac{500}{3} \right)$$

$$= \boxed{\pi \left(\frac{1000}{3} \right)}$$

Note: If you did not solve for y and revolved about the x -axis, the integral would be $\int_0^{10} \pi \left(\sqrt{25 - \frac{x^2}{4}} \right) dx$
 $= \frac{500\pi}{3} \neq \frac{1000\pi}{3}$.

It is important to revolve about the correct axis.