QUIZ 10: LESSONS 17-18
MARCH 1, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. Suppose an ant is walking to his colony 1 foot away and every minute he travels a quarter of the distance he traveled in the previous minute. Assume that in the first minute he travels half a foot.

   (a) [3 pts] Write an infinite series that describes how far the ant travels given an infinite amount of time.

   Solution: We are told that at minute $t = 1$, he travels $\frac{1}{2}$ of a foot. But at minute $t = 2$, he only travels a quarter of that distance, which is $\frac{1}{4}\left(\frac{1}{2}\right)$ feet. At minute $t = 3$, he travels a quarter of *that* distance, which is $\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\right)\right) = \left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)$ feet. Hence, the series we want is

   $$\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{1}{4}\right)^n.$$

   (b) [1 pts] Will the ant ever make it back to his colony? (Answer yes or no.)

   Solution: The ant will make it back to his colony if $\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{1}{4}\right)^n = 1$.

   However,

   $$\sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{1}{4}\right)^n = \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{2}\left(\frac{1}{\frac{3}{4}}\right) = \frac{1}{2}\left(\frac{4}{3}\right) = \frac{2}{3} \neq 1.$$

   So **no**, the ant will not make it back to his colony.

2. [3 pts] Find the domain of

   $$\frac{1}{x\ln(y+1)}.$$

   Write your answer in set builder notation.

   Solution: Since we can't have the bottom equal to zero, we can't have $x = 0$ or $\ln(y+1) = 0$. Hence, $x \neq 0$ and $y \neq 0$ (if $y = 0$, then $\ln(y+1) = \ln(1) = 0$). We also need $\ln(y+1)$ to exist. So we must have $y + 1 > 0$, which means $y > -1$. In set builder notation, this is

   $$\{(x, y) : x \geq 0, y \neq 0, y > -1\}.$$

3. [3 pts] Sketch a graph of the level curve of

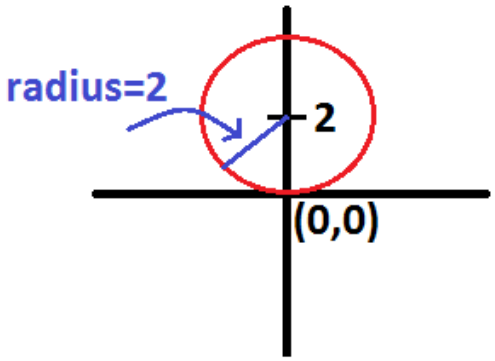
$$f(x,y) = \ln(x^2 + (y-2)^2).$$

for $z = \ln 4$. Label important point(s) of your graph.

Solution: We are given a specific point to investigate, so we simplify $\ln 4 = \ln(x^2 + (y-2)^2)$. Applying e to both sides, we get

$$4 = x^2 + (y-2)^2.$$

This is a circle of radius 2 centered at $(0,2)$. So our graph looks like



Comments 0.1.

(1) The terms in a summation are being indexed by n. Writing something like

$$\sum_{n=0}^{\infty} \frac{1}{2} + \left(\frac{1}{4}\right)^n$$

means the sum actually looks like

$$\left[\frac{1}{2} + \left(\frac{1}{4}\right)^0\right] + \left[\frac{1}{2} + \left(\frac{1}{4}\right)^1\right] + \left[\frac{1}{2} + \left(\frac{1}{4}\right)^2\right] + \cdots.$$

So this summation means $\frac{1}{2}$ is added an infinite amount of times. If you mean for $\frac{1}{2}$ to be added just once, you ought to write

$$\frac{1}{2} + \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n.$$

(2) Series and improper integrals aren't the same thing. If you are asked to write an infinite series to describe a situation and you start writing down an integral, you've done something wrong.