QUIZ 12: LESSONS 20-21
MARCH 10, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the second order derivatives of

$$f(x,y) = \frac{x+y}{y}.$$

Solution: We start with taking the derivatives with respect to x. Write

$$\begin{aligned} f_x(x,y) &= \frac{\partial}{\partial x}\left(\frac{x+y}{y}\right) \\ &= \frac{1}{y}\frac{\partial}{\partial x}(x+y) \\ &= \frac{1}{y}(1) = \frac{1}{y}. \end{aligned}$$

Thus,

$$f_{xx}(x,y) = 0 \text{ and } f_{xy} = -\frac{1}{y^2}.$$

Next, we need to take the derivative with respect to y. Write

$$\begin{aligned} f_y(x,y) &= \frac{\partial}{\partial y}\left(\frac{x+y}{y}\right) \\ &= \frac{y\frac{\partial}{\partial y}(x+y) - (x+y)\frac{\partial}{\partial y}(y)}{y^2} \\ &= \frac{y(1) - (x+y)(1)}{y^2} \\ &= \frac{-x}{y^2} \end{aligned}$$

Thus,

$$\begin{aligned} f_{yy}(x,y) &= \frac{\partial}{\partial y}\left(\frac{-x}{y^2}\right) \\ &= \frac{x}{2y^3}. \end{aligned}$$

2. [5 pts] Suppose a box with a square base initially has a width of 5 in and height of 10 in. What is the change in the volume of the box if we increase the width to 6 in and decrease the height to 9 in?

Solution: The volume of the box is given by $V = w^2h$ because we have assumed that the box has a square base. By our approximation of ΔV, we have

$$\Delta V \approx \frac{\partial V}{\partial w}(\Delta w) + \frac{\partial V}{\partial h}(\Delta h).$$

Since

$$\frac{\partial V}{\partial w} = 2wh \text{ and } \frac{\partial V}{\partial h} = w^2,$$

we can write

$$\begin{aligned}\Delta V &\approx 2wh(\Delta w) + w^2(\Delta h)\\ &= 2(5)(10)(1) + (5)^2(-1)\\ &= 100 - 25 = 75.\end{aligned}$$