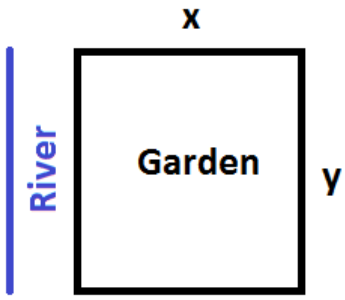
QUIZ 15: LESSON 25-26
MARCH 31, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

Suppose you are planning to build a rectangular garden with the river on one side that you want to enclose on the other 3 sides. Assume you have 500 ft of fencing and are asked to maximize the area of the garden using Lagrange multipliers.

1. [1 pt] Label your variables

   Solution:



   So, x is our width and y is our length.

2. [2 pts] Label your functions f and g

   Solution: Because we are asked to maximize area, we must have $f(x, y) = xy$. Further, $g(x, y) = 2x + y$ because we are told we only have 500 feet of fencing and we don't want fencing along the river.

3. [3 pts] Write down the system of equations you need to solve

   Solution: Differentiating, we get

$$f_x = y, f_y = x, g_x = 2, g_y = 1.$$

   Thus, the system of equations we need to solve is

$$y = 2\lambda$$
$$x = \lambda$$
$$2x + y = 500.$$

4. [3 pts] Find all solutions to the system of equations in # 3

   Solution: We see that since $\lambda = x$, $y = 2\lambda = 2x$. Thus, by our constraint

$$\begin{aligned} & 2x + y = 500 \\ \Rightarrow \quad & 2x + 2x = 500 \\ \Rightarrow \quad & 4x = 500 \\ \Rightarrow \quad & x = 125 \\ \Rightarrow \quad & y = 2x = 2(125) = 250. \end{aligned}$$

   Hence, our only solution is

$$(x, y) = (125, 250).$$

5. [1 pts] What is the maximum area of the garden?

   Solution: The maximum area is given by $f(125, 250) = 125(250) = 31,250$.