QUIZ 16: LESSON 27
APRIL 3, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Evaluate

$$\int_0^1 \int_0^2 4x^3 y \; dy \, dx.$$

Solution: We integrate first with respect to y, which means that we treat $4x^3$ as a constant:

$$\begin{aligned}
\int_0^1 \int_0^2 4x^3 y \; dy \, dx &= \int_0^1 \left(\frac{4x^3}{2} \right) y^2 \Big|_{y=0}^{y=2} dx \\
&= \int_0^1 2x^3 \underbrace{\left[(2)^2 - (0)^2 \right]}_{4} dx \\
&= \int_0^1 8x^3 \, dx \\
&= 2x^4 \big|_0^1 \\
&= 2(1)^4 - 2(0)^4 \\
&= \boxed{2}
\end{aligned}$$

2. [5 pts] Evaluate

$$\int_1^e \int_0^{1/x} \ln x \; dy \, dx.$$

Solution: Again, we integrate with respect to y and treat $\ln x$ as a constant. Write

$$\begin{aligned}
\int_1^e \int_0^{1/x} \ln x \; dy \, dx &= \int_1^e y \ln x \Big|_{y=0}^{y=1/x} dx \\
&= \int_1^e \left[\frac{1}{x} \ln x - (0) \ln(x) \right] dx \\
&= \int_1^e \left(\frac{1}{x} \right) \ln x \, dx.
\end{aligned}$$

This is now a u-substitution problem. Take $u = \ln x$, then $du = \dfrac{1}{x}dx$. Further, $u(1) = \ln(1) = 0$ and $u(e) = \ln e = 1$ so

$$\begin{aligned}\int_1^e \left(\frac{1}{x}\right) \ln x \, dx &= \int_{u(1)}^{u(e)} \underbrace{(\ln x)}_{u} \underbrace{\left(\frac{1}{x}\, dx\right)}_{du} \\ &= \int_0^1 u \, du \\ &= \frac{1}{2}u^2\Big|_0^1 \\ &= \boxed{\frac{1}{2}}\end{aligned}$$