QUIZ 20: LESSONS 33-34
APRIL 21, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. Find the determinants of the following matrices. If the determinant is non-zero, find the inverse of the matrix.

   (a) [3 pts]

$$\begin{bmatrix} -1 & 2 \\ -4 & 8 \end{bmatrix}$$

Solution:

$$\begin{vmatrix} -1 & 2 \\ -4 & 8 \end{vmatrix} = -1(8) - (-4)(2) = -8 + 8 = 0$$

Because the determinant is zero, this matrix is singular and hence **does not** have an inverse.

   (b) [3 pts]

$$\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$

Solution:

$$\left[\begin{matrix} 1 & 1 \\ 2 & -2 \end{matrix}\right| = 1(-2) - (2)(1) = -4$$

Because the determinant is non-zero, this matrix is non-singular and hence **does** have an inverse. We use row operations to find its inverse:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & -2 & 0 & 1 \end{array}\right] \overset{-2R_1+R_2\to R_2}{\longrightarrow} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -4 & -2 & 1 \end{array}\right]$$

$$\overset{-\frac{R_2}{4}\to R_2}{\longrightarrow} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 1/2 & -1/4 \end{array}\right]$$

$$\overset{-R_2+R_1\to R_1}{\longrightarrow} \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 1/4 \\ 0 & 1 & 1/2 & -1/4 \end{array}\right].$$

The inverse is then

$$\begin{bmatrix} 1/2 & 1/4 \\ 1/2 & -1/4 \end{bmatrix}.$$

We could also use the following formula:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Then, for $A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$ we would get

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} -2 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & -1/4 \end{bmatrix}.$$

2. [4 pts] Is the matrix

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

singular or non-singular? Give a reason for your answer.

Solution: We say a matrix is singular if its determinant is 0. So we find the determinant of A. We will use the minor and cofactor method although it is perfectly fine to use the trick as well. We will expand along the 2^{nd} row because there's a zero there.

(2,1)-entry:

$$\begin{aligned} M_{21} &= \begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} \\ &= (-1)(1) - (-1)(1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} C_{21} &= (-1)^{2+1} M_{21} \\ &= 0 \end{aligned}$$

(2,2)-entry:

$$\begin{aligned} M_{22} &= \begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} \\ &= 1(1) - (-2)(-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} C_{32} &= (-1)^{2+2} M_{32} \\ &= -1 \end{aligned}$$

(2,3)-entry:

$$\begin{aligned} M_{23} &= \begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} \\ &= 1(1) - (-2)(-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} C_{33} &= (-1)^{2+3} M_{33} \\ &= 1 \end{aligned}$$

Hence,

$$\det(A) = \underset{\substack{\uparrow \\ (2,1)\text{-} \\ \text{entry}}}{(0)} C_{21} + \underset{\substack{\uparrow \\ (2,2)\text{-} \\ \text{entry}}}{(2)} C_{22} + \underset{\substack{\uparrow \\ (2,3)\text{-} \\ \text{entry}}}{(1)} C_{23} = 0(0) + 2(-1) + 1(1) = -1.$$

Because this is non-zero, A is **non-singular**.