Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Evaluate

$$\int xe^{-x}\, dx.$$

Solution: This is an integration by parts problem. By LIATE, we take $u = x$. Then

$$\begin{aligned} u &= x & dv &= e^{-x}\, dx \\ du &= dx & v &= -e^{-x} \end{aligned}$$

Hence,

$$\int xe^{-x}\, dx = -xe^{-x} - \underbrace{\int (-e^{-x})\, dx}_{(\star)}$$

$$= -xe^{-x} - e^{-x} + C$$

$$= -(x+1)e^{-x} + C.$$

To compute $(\star)$, we use u-substitution. Let $u = -x$, then $du = -dx$. So

$$\int -e^{-x}\, dx = -\int (-e^{u})\, du = \int e^{u}\, dx = e^{u} + C = e^{-x} + C.$$

2. [4 pts] Evaluate

$$\int_{e}^{e^{16}} \frac{1}{x \ln x}\, dx.$$

Solution: This is a u-substitution problem, although you could use integration by parts. Let $u = \ln x$, then $du = \frac{1}{x}\, dx$. Moreover, $u(e) = \ln e = 1$ and $u(e^{16}) = \ln e^{16} = 16$. Putting this together,

$$\int_{e}^{e^{16}} \frac{1}{x \ln x}\, dx = \int_{1}^{16} \frac{1}{u}\, du$$

$$= \ln |u| \Big|_{1}^{16}$$

$$= \ln 16 - \ln 1$$

$$= \ln 16.$$

3. [2 pts] Suppose you are given the integral

$$\int x^7 \ln x^3 \, dx$$

and you are told to use integration by parts. What should you choose for your u?

<u>Solution</u>: If you rewrite the integral as

$$\int x^7 \ln x^3 \, dx = 3 \int x^7 \ln x \, dx,$$

then by LIATE you should take $u = \ln x$. Then $du = \frac{1}{x}$.

If you decide not to simplify the integral, you can also choose $u = \ln x^3$. You would then just need to note that

$$du = \frac{3x^2}{x^3} = 3 \left(\frac{1}{x} \right).$$

There is nothing wrong with choosing this, it simply means you need to be more careful when taking the derivative. For example, $du \neq \frac{1}{x^3}$ and $du \neq \frac{1}{x}$ in this case.