QUIZ 4 SOLUTIONS: LESSONS 5-6
JANUARY 27, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Evaluate

$$\int_0^{\pi/2} x \cos x \, dx.$$

Solution: This is an integration by parts problem. By LIATE, we need $u = x$. So

$$\begin{array}{ll} u = x & dv = \cos x \, dx \\ du = dx & v = \sin x \, dx \end{array}$$

Hence,

$$\begin{aligned} \int_0^{\pi/2} x \cos x \, dx &= x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx \\ &= x \sin x \Big|_0^{\pi/2} - (-\cos x) \Big|_0^{\pi/2} \\ &= (x \sin x + \cos x) \Big|_0^{\pi/2} \\ &= \left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} + \cos \frac{\pi}{2} - (0 \sin 0 + \cos 0) \\ &= \left(\frac{\pi}{2}\right) (1) + 0 - 1 \\ &= \frac{\pi}{2} - 1. \end{aligned}$$

2. [5 pts] Let $P(t)$ be the mass of a radioactive substance after t years. If $P'(t) = -10P(t)$, find the half-life of the substance.

Solution: There are two ways you can do this. The first way is to remember that if $P'(t) = kP(t)$, then half-life is given by the formula

$$\text{half-life} = \frac{\ln \frac{1}{2}}{k}.$$

Hence, our answer is $-\frac{1}{10} \ln \frac{1}{2}$.

A more honest way of doing this problem is working through all the steps. We are given

$$P'(t) = -10P(t).$$

So we write

$$\begin{aligned} \frac{dP}{dt} &= -10P \\ \Rightarrow \quad P\,dP &= -10\,dt \\ \Rightarrow \quad \int \frac{1}{P}\,dP &= \int (-10)\,dt \\ \Rightarrow \quad \ln|P| &= -10t + C \\ \Rightarrow \quad e^{\ln|P|} &= e^{-10t+C} \\ \Rightarrow \quad P &= e^{-10t+C} \\ \Rightarrow \quad P &= e^{C}e^{-10t} \end{aligned}$$

Now, we are looking for the t such that

$$\frac{P(0)}{2} = e^{C}e^{-10t}$$

where $P(0)$ is the initial amount of the substance. Observe that

$$P(0) = e^{C}e^{-10(0)} = e^{C}.$$

Thus,

$$\frac{P(0)}{2} = P(0)e^{-10t},$$

which implies

$$e^{-10t} = \frac{1}{2}.$$

Applying ln to both sides, we get

$$-10t = \ln\frac{1}{2} \Rightarrow t = \frac{\ln\frac{1}{2}}{-10}.$$

3. [1 pt] **True or False**: Is the given function separable?

$$y^2t + t + y$$

False: You can't write this in the form $f(t)g(y)$ because no matter how you try to separate the variables, you'll have some mixture of the t and y.