QUIZ 6: LESSONS 9-10
FEBRUARY 8, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the particular solution to

$$\frac{dy}{dx} + \frac{y}{x} = x \text{ when } y(1) = 0.$$

Solution: We first need to find the general solution and then use the initial conditions to determine the unknown.

Because of the form the differential equation is in, we use the FOLDE method. So we go through the steps.

Step 1: Find P, Q

$$P(x) = \frac{1}{x}, \quad Q(x) = x.$$

Step 2: Find integrating factor.

Write

$$e^{\int P(x)\,dx} = e^{\int \frac{1}{x}\,dx}$$

We know $\int \frac{1}{x}\,dx = \ln|x| + C$ and for an integrating factor we always assume $C = 0$, hence

$$e^{\int \frac{1}{x}\,dx} = e^{\ln|x|} = |x|.$$

Now, we assume that $x > 0$ so we can drop the absolute value symbols. Hence our integrating factor is

$$u(x) = x.$$

Step 3: Set up equation

Our equation is

$$yu(x) = \int Q(x)u(x)\,dx.$$

Plugging in what we know, we have

$$yx = \int x \cdot x\,dx = \int x^2\,dx = \frac{1}{3}x^3 + C$$

Thus,

$$y = \frac{1}{3}x^2 + \frac{C}{x}.$$

We are given $y(1) = 0$, so

$$0 = \frac{1}{3}(1)^2 + \frac{C}{1} = \frac{1}{3} + C.$$

Hence $C = -\frac{1}{3}$. Thus, our solution is

$$y = \frac{1}{3}x^2 - \frac{1}{3x}.$$

2. Consider the following differential equation:

$$ty' + t^7 y = 13t.$$

(a) [2 pts] Find P and Q (clearly label each, else I will take off a point).

Solution: Our equation is not in the correct form to find P and Q. We first divide both sides by t. So our equation becomes

$$y' + t^6 y = 13.$$

Hence,

$$P(t) = t^6, \quad Q(t) = 13.$$

(b) [3 pts] Find the integrating factor.

Solution: Our integrating factor is given by

$$e^{\int P(t)\,dt} = e^{\int t^6\,dt} = e^{\frac{1}{7}t^7}.$$