QUIZ 7: LESSONS 11-12
FEBRUARY 13, 2017

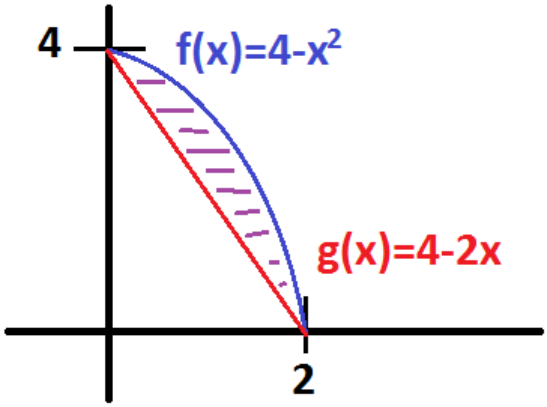
Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers and sub-parts, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. Consider the region enclosed by the curves

$$y = 4 - x^2, \quad y = 4 - 2x.$$

(a) [3 pts] Sketch the graph of the region described above. Label your axes, functions, and points of intersection.

Solution:



(b) [2 pts] Setup (but do **not** evaluate) the integral that describes the area of the region above.

Solution: By our sketch, our points of intersection are $x = 0$ and $x = 2$. Hence, our integral is given by

$$\int_0^2 [\underbrace{(4 - x^2)}_{\substack{\text{top} \\ \text{function}}} - \underbrace{(4 - 2x)}_{\substack{\text{bottom} \\ \text{function}}}]\, dx = \int_0^2 (2x - x^2)\, dx.$$

(c) [2 pts] Evaluate the integral from part (b).

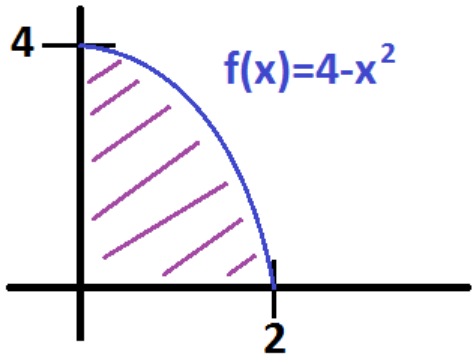
Solution: Write

$$\int_0^2 (2x - x^2)\, dx = \frac{2}{2}x^2 - \frac{x^3}{3}\Big|_0^2$$

$$= (2)^2 - \frac{2^3}{3}$$
$$= 4 - \frac{8}{3}$$
$$= \frac{12}{3} - \frac{8}{3}$$
$$= \frac{4}{3}.$$

2. [3 pts] Consider the region in the first quadrant bounded by $y = 4 - x^2$. Setup (but do **not** evaluate) the integral that represents the volume of this region obtained by revolving this region about the y-axis.

Solution: By our work in problem 1, we know that our region looks like



Observe that we are only asked to consider the region in the first quadrant. Since this is about the y-axis, we need to rewrite our function in the form of $x =$something in y. So

$$y = 4 - x^2$$
$$\Rightarrow \quad y + x^2 = 4$$
$$\Rightarrow \quad x^2 = 4 - y$$
$$\Rightarrow \quad x = \sqrt{4 - y}$$

We have only taken the positive root because we are in the first quadrant and so $x \geq 0$. So, our integral is

$$\int_0^4 \pi \left(\sqrt{4 - y}\right)^2 \, dy = \int_0^4 \pi(4 - y) \, dy.$$