QUIZ 9 SOLUTIONS: LESSONS 15-16
FEBRUARY 24, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Determine if the integral converges or diverges. If it converges, find its value.

$$\int_1^\infty \frac{3}{x^3}\,dx.$$

Solution: Write

$$\int_1^\infty \frac{3}{x^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{3}{x^3}\,dx.$$

We first compute the integral and then take the limit.

$$\begin{aligned}\int_1^t \frac{3}{x^3} &= -\frac{3}{2x^2}\Big|_1^t \\ &= -\frac{3}{2t^2}+\frac{3}{2}.\end{aligned}$$

Now that we know the integral, we can take the limit:

$$\int_1^\infty \frac{3}{x^3}\,dx = \lim_{t\to\infty}\left(-\frac{3}{2t^2}+\frac{3}{2}\right) = \frac{3}{2}$$

since $\lim_{t\to\infty}\left(-\frac{3}{2t^2}\right) = 0$. Thus the integral converges and its value is $\frac{3}{2}$.

2. [5 pts] Compute

$$\sum_{n=1}^{\infty}\frac{3}{4^{n+1}}.$$

Solution: This is a geometric series so we want to apply the geometric series formula. However, this is not quite in the right form because the index does not begin at 0. Write

$$\sum_{n=1}^{\infty}\frac{3}{4^{n+1}} = \sum_{n=0}^{\infty}\frac{3}{4^{n+2}}.$$

Now, that the index is correct we have to address that the exponent is $n+2$ not n. Write

$$\sum_{n=0}^{\infty}\frac{3}{4^{n+2}} = \sum_{n=0}^{\infty}\frac{3}{4^2 4^n}.$$

Finally, we see that $c = \frac{3}{4^2} = \frac{3}{16}$ and $r = \frac{1}{4}$ (where we note that $|r| < 1$). Hence,

$$\sum_{n=1}^{\infty} \frac{3}{4^{n+1}} = \sum_{n=0}^{\infty} \frac{3}{16} \left(\frac{1}{4}\right)^n = \frac{\frac{3}{16}}{1 - \frac{1}{4}} = \frac{3}{16 - 4} = \frac{3}{12} = \frac{1}{4}.$$