MATH 265 Credit Examination

Name : ______________________
ID : ______________________

No books, notes, or calculators allowed. Please be neat and show all your work. Circle your answers. Return this entire booklet

Problem Number	Possible Points	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
SUBTOTAL	120	
I	20	
II	20	
III	20	
IV	20	
SUBTOTAL	80	
TOTAL	200	

These short response questions are worth 10 points each. Unless otherwise noted in brackets [x], each part is worth 2 points. **Some problems may have more than one correct answer.** On questions with several correct answers, your score will be computed on the basis of the number of your correct responses minus the number of your incorrect responses. In general only answers will be graded and no partial credit given.

1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

(a) [5] If possible, find the inverse A^{-1}.

(b) [5] Display the reduced row echelon form of the matrix A.

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2. Which of the following are always true? Circle the correct responses.

(a) Every 5 linearly independent vectors in $\mathbf{R}^5$ form a basis of $\mathbf{R}^5$. Yes No

(b) Every 4 orthonormal vectors in $\mathbf{R}^6$ are linearly independent. Yes No

(c) The rows of a 5×4 matrix are linearly dependent. Yes No

(d) If A is a 4×4 matrix of rank 3, then the system of equations $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Yes No

(e) $\mathbf{R}^4$ can be spanned by 6 vectors. Yes No

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3. The plane described by the solutions to the equation

$$2x - 2y + 8z = 0$$

is a subspace of $\mathbf{R}^3$. Circle each set of vectors below that spans this subspace.

(i) $\{(-4, 0, 1),\ (-3, 1, 1),\ (1, 1, 0)\}$

(ii) $\{(0, 4, 1),\ (0, 8, 2)\}$

(iii) $\{(2, -2, 8)\}$

(iv) $\{(-3, 1, 1),\ (-2, 2, 1)\}$

(v) $\{(4, 0, 1),\ (-2, 2, 1)\}$

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4. Consider the vectors $\mathbf{v}_1 = (0, 2, -2, 1)$ and $\mathbf{v}_2 = (1, -1, 1, 2)$ in $\mathbf{R}^4$. Use the Gram-Schmidt process (in the order given) to produce an **orthogonal** basis for the span of $\mathbf{v}_1$ and $\mathbf{v}_2$.

First basis vector = ____________

Second basis vector = ____________

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5. Let W be a two-dimensional subspace of $\mathbf{R}^3$. Suppose that its orthogonal complement $W^\perp$ is spanned by $(2, 0, 1)$. For the given vector $\mathbf{u} = (5, 5, -15)$ find the (unique) vectors $\mathbf{w}$ in W, and $\mathbf{v}$ in $W^\perp$, such that $\mathbf{u} = \mathbf{w} + \mathbf{v}$.

$\mathbf{w} =$ ____________ , $\mathbf{v} =$ ____________.

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6. Circle every set of vectors below that is a basis of $\mathbf{R}^3$.

(i) $\{ (1, 1, 1), (1, 3, 6) \}$

(ii) $\{ (3, 0, 2), (0, 1, 0), (0, 0, -1) \}$

(iii) $\{ (1, 0, 0), (1, 1, 1), (0, 4, 5) \}$

(iv) $\{ (1, 2, 5), (2, 3, 12), (0, 1, 1), (1, 0, 1) \}$

(v) $\{ (2, 3, 4), (-2, 5, 0), (2, 1, 3) \}$

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7. Which of the following are always true? Circle the correct responses.

(a) $\det(AB) = \det(B)\det(A)$. Yes No

(b) $\det(A + 2B) = \det(A) + 2\det(B)$. Yes No

(c) If $A\mathbf{x} = 0$ has a non-trivial solution, then $\det(A) = 0$. Yes No

(d) If the nullity of A is 0, then $\det(A) = 0$. Yes No

(e) If A is invertible, then $\mathbf{rref}(A) = I$. Yes No

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8. The system of equations

$$\begin{array}{rcrcrcrcr} x & + & 2y & - & z & & & = & 5 \\ 4x & + & 5y & - & z & + & 4w & = & 10 \\ x & + & y & & & + & w & = & 2 \end{array}$$

has an augmented matrix which is row equivalent to the matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

Which of the following is a possible general solution to this system of equations?

(i) $(1, 2, 0, -1)$

(ii) $(0, 3, 1, -1) + t(-2, 2, 2, 0)$

(iii) $(1, 2, 0, -1) + t(-1, 1, 1, 0)$

(iv) $(1, 2, 1, -1) + t(-1, 1, 1, 0)$

(v) $(1, 0, 1, 0) + t(0, 1, -1, 0) + s(0, 0, 0, 1)$

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9. Let A and B be similar matrices, that is, $B = P^{-1}AP$ for some invertible matrix P. Which of the following are always true? Circle the correct responses.

(a)	The matrix P must be unique.	Yes	No
(b)	The matrix B must be diagonal.	Yes	No
(c)	There is an invertible matrix Q such that $A = Q^{-1}BQ$.	Yes	No
(d)	The matrices A and B must have the same eigenvectors.	Yes	No
(e)	The matrices A and B must have the same determinants.	Yes	No

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10. If $A = \begin{bmatrix} k^2 & 0 & k \\ -1 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix}$, for which values of k does the system of equations $A\mathbf{x} = \mathbf{0}$ have a non-zero solution? Circle the correct responses.

(i) all $k \leq 1$;

(ii) $k = 0,\ 1$;

(iii) $k = 0,\ -1$;

(iv) all $k \neq 0$;

(v) $k = 1,\ 0,\ -1$.

11. Suppose A is an $n \times n$ invertible matrix and $\mathbf{b}$ is a vector in $\mathbf{R}^n$. Which of the following are always true? Circle the correct responses.

(i) $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.	Yes	No
(ii) $\text{rank}(A) = n$.	Yes	No
(iii) The nullity of A is n.	Yes	No
(iv) $\mathbf{b}$ is in the column space of A.	Yes	No
(v) $A\mathbf{x} = \mathbf{b}$ has only one solution.	Yes	No

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12. The vector function $\mathbf{x}(t) = e^{-t}(1, -1) + e^{3t}(1, 1)$ is a solution to the system of linear differential equations

$$\mathbf{x}'(t) = A\mathbf{x}(t)$$

for a fixed unknown 2×2 matrix A.

(a) [3] Find the eigenvalues of the matrix A.

(b) [3] Find the eigenvectors of the matrix A.

(c) [4] Find the matrix A.

No notes, books, or calculators allowed. Show all work in the exam pages and hand the entire examination back at the end of the two hours. These discussion problems are worth 20 points each with some partial credit. **For full credit, all work must be shown.** Circle the answer. If you have questions about what is required, ask the instructor.

I. The matrix $A = \begin{bmatrix} 1 & -2 & 0 & 3 & -4 \\ 3 & 2 & 8 & 1 & 4 \\ 2 & 4 & 8 & -2 & 8 \\ -1 & 2 & 0 & 4 & -3 \end{bmatrix}$ has the reduced row echelon form

$$\mathbf{rref}(A) = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) [4] Find a basis for the rowspace of A.

(b) [4] Find a basis for the columnspace of A.

(c) [4] Find a basis for the nullspace of A.

(d) [4] Find a non-zero vector $\mathbf{v}$ which is perpendicular to the rowspace of A.

(e) [4] Compute the rank and the nullity of A.

II. Let $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(a) [6] Find the eigenvalues of B.

(b) [6] Find two linearly independent eigenvectors of B.

(c) [5] Find, if possible, an invertible matrix P such that the matrix $D = P^{-1}BP$ is diagonal.

(d) [3] Find, if possible, an orthogonal matrix Q such that the matrix $E = Q^T BQ$ is diagonal.

III. Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$.

(a) [5] Find the eigenvalues of A.

(b) [5] Find two linearly independent eigenvectors of A.

(c) [5] Write the general solution to the system of differential equations

$$\mathbf{x}'(t) \equiv \frac{d\mathbf{x}}{dt} = A\mathbf{x}(t).$$

(d) [5] Find the solution $\mathbf{x}(t)$ that satisfies the initial condition $\mathbf{x}(0) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$.

IV. Let W be the column space of the matrix $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \\ -1 & 3 \\ 3 & 1 \end{bmatrix}$, and let $\mathbf{b} = \begin{bmatrix} 0 \\ 7 \\ 7 \\ 0 \end{bmatrix}$.

(a) [3] Show that $\mathbf{b}$ is not on the subspace W.

(b) [7] Compute the orthogonal projection of $\mathbf{b}$ onto W.

(c) [5] Compute the distance from $\mathbf{b}$ to W.

(d) [5] Find the least squares solution $\mathbf{z}$ to $A\mathbf{x} = \mathbf{b}$.