

1. Consider the initial value problem $y' = x^2y - xy^2$, $y(2) = 3$.

(a) Evaluate $y'(2)$.

(b) Is the solution increasing or decreasing near $x = 2$?

(c) Evaluate $y''(2)$.

(d) Is the solution concave upward
or concave downward near $x = 2$?

(e) Are the Euler tangent line approximations of the solution near $x = 2$
greater than or less than the values of the solution?

2. Find the first three nonzero terms of the Maclaurin series of the solution
of the initial value problem $y' = xy$, $y(0) = 1$.

3. Find the first four nonzero terms of the Maclaurin series of the solution of the initial value problem $y'' - 2y' + y = 0$, $y(0) = 0$, $y'(0) = 1$.

4. Find the first four nonzero terms of the Taylor series about $c = 1$ of the solution of the initial value problem $y' = y^2$, $y(1) = 1$.

1. (a) Find the solution of the initial value problem $y' + 2y = e^{-t}$, $y(0) = 1$.

(b) What is the value of the solution at $t = 1$?

(c) What is the smallest value of n for which the Euler Tangent Line Method with n steps gives an approximate value of the solution that is within 0.05 of the actual solution at $t = 1$?

2. (a) Find the solution of the initial value problem $y' - 2y = -3e^{-t}$, $y(0) = 1$.

(b) What is the value of the solution at $t = 1$?

(c) What is the smallest value of n for which the Euler Tangent Line Method with n steps gives an approximate value of the solution that is within 0.05 of the actual solution at $t = 1$?

3. (a) Use the given direction field of the differential equation $y' + 2y = e^{-t}$ to plot the solutions that satisfy the initial conditions $y(0) = 0.95$, $y(0) = 1$, and $y(0) = 1.05$.

(b) Use the given direction field of the differential equation $y' - 2y = -3e^{-t}$ to plot the solutions that satisfy the initial conditions $y(0) = 0.95$, $y(0) = 1$, and $y(0) = 1.05$.

(c) Refer to the plots in (a) and (b) above to briefly explain why more terms were required to obtain the same degree of accuracy using the Euler Tangent Line Method in Problem 2 than in Problem 1.

1. Find the solution of the initial value problem $y' = 2y - 1$, $y(0) = 1$.
 $\phi(t) =$ _____

Find the approximate value of the solution of the initial value problem $y' = 2y - 1$, $y(0) = 1$, where $t = 0.4$ using:

- the Euler method (eul) with $h = 0.1$ _____
- the Euler method (eul) with $h = 0.05$ _____
- the Euler method (eul) with $h = 0.025$ _____
- the improved Euler method (rk2) with $h = 0.1$ _____
- the Runge–Kutta method (rk4) with $h = 0.1$ _____
- the solution $\phi(t)$ _____

2. Find the approximate value of the solution of the initial value problem $y' = \sqrt{t+y}$, $y(1) = 3$, where $t = 2$ using :

- the Euler method (eul) with $h = 0.025$ _____
- the Euler method (eul) with $h = 0.0125$ _____
- the improved Euler method (rk2) with $h = 0.1$ _____
- the improved Euler method (rk2) with $h = 0.05$ _____
- the Runge–Kutta method (rk4) with $h = 0.2$ _____
- the Runge–Kutta method (rk4) with $h = 0.1$ _____

3. Give reasons why the Euler tangent line method with $h = 0.1$ does not give a good approximation of the value of the solution of the initial value problem where $t = 1$.

(a) $y' = (y + 1.25)^2$, $y(0) = 0$,

solution $y = \frac{25t}{4(4 - 5t)}$.

(b) $y' = \frac{50t}{64(1 - 2y)}$, $y(0) = 0$,

solution $y = \frac{1 - \sqrt{1 - 25t^2/16}}{2}$.

(c) $y' = 2(ty)^{1/3}$, $y(0) = 0$,

solution $y = t^2$.

(d) $y' = 4e^{-t} - 3(1 - y)$, $y(0) = 0$,

solution $y = 1 - e^{-t}$.