Answers to practice problems

Caution. This is a preliminary draft. Use at your own risk. Ask your instructor for better information on the answers.

1. (a) $y = C_1 e^t + C_2 e^{2t}$ (b) $y = C_1 e^{3t} + C_2 t e^{3t}$ (c) $y = C_1 e^{-t}\cos t + C_2 e^{-t}\sin t$

2. $y = C_1 + C_2 t + C_3 t^2 + \dfrac{t^3}{6}$ **3.** $y = \dfrac{t^3}{6} + \dfrac{11}{6t^3}$

4. (i) LINEAR, HOMOG, EXACT (ii) LINEAR, SEPARABLE (iii) HOMOG **5.** B

6. $y_p = At^2e^t + B\cos t + C\sin t + e^{2t}(Dt+E)$ **7.** $y_p = \frac{1}{2}\sin 2t - \cos 2t$

8. $\frac{dQ}{dt} = 10 - \frac{2Q}{100+3t}$, $Q(0) = 20$; $Q(t) = 200 + 6t - 180(1 + \frac{3t}{100})^{-2/3}$

9. $y(0.5) \approx 0.6875$ **10.** $-0.5 < t < 1.5$ (see diagram below) **11.** $y_2 = t^{-3}$

12. (a) $u = e^{-\frac{t}{2}}\sin t$ (b) E **13.** (a) $\dfrac{1}{s} + \dfrac{60}{s^4} - \dfrac{2}{s^2+4}$ (b) $\dfrac{e^{15-5s}}{s-3}$

14. (a) $\dfrac{1}{3} + \dfrac{2}{3}e^{-3t}$ (b) $\frac{7}{2}u_3(t)\sin 2(t-3)$ **15.** $y = \frac{1}{2}u_3(t)\sin 2(t-3) + \frac{3}{4}\cos 2t + \frac{1}{4}$

16. Let $x_1 = y, x_2 = y'$, then $\begin{cases} x_1' = x_2 \\ x_2' = -e^{-t}x_1 + 2tx_2 + 1 \end{cases}$

17. $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ **18.** $\vec{\mathbf{x}}(t) = e^t \begin{pmatrix} 2\sin 2t + \cos 2t \\ \cos 2t - \frac{1}{2}\sin 2t \end{pmatrix}$

19. $\vec{\mathbf{x}}_p(t) = \dfrac{e^{2t}}{3}\begin{pmatrix} -2 \\ -8 \end{pmatrix}$

20. (i) one positive, one negative eigenvalue, so is a saddle point. So has to be C or F. Look at limit as t gets large. Most of the trajectories approach the line through (2,1) so the answer is C.
ii) both eigenvalues are positive, so is A or D. As t gets large, the exp(2t) term dominates, so trajectories approach the (2,1) line. Thus the answer is D .
(iii) The eigenvalues are complex, so the trajectories should "swirl". Thus the answer is G or H. Plugging in t= 0 and $c_1 = 1, c_2 = 0$ shows that the direction of the swirl is like that of H. So the answer is H.