

ANSWERS TO MATH 266 FINAL PRACTICE PROBLEMS

Caution. This is a preliminary draft. Use at your own risk. Ask your instructor for better information on the answers.

1. (a) $y = C_1 e^t + C_2 e^{2t}$ (b) $y = C_1 e^{3t} + C_2 t e^{3t}$ (c) $y = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t$
2. $y = C_1 + C_2 t + C_3 t^2 + \frac{t^3}{6}$ 3. $y = \frac{t^3}{6} + \frac{11}{6t^3}$
4. (i) LINEAR, HOMOG, EXACT (ii) LINEAR, SEPARABLE (iii) HOMOG 5. B
6. $y_p = At^2 e^t + B \cos t + C \sin t + e^{2t}(Dt + E)$ 7. $y_p = \frac{1}{2} \sin 2t - \cos 2t$
8. $\frac{dQ}{dt} = 10 - \frac{2Q}{100+3t}$, $Q(0) = 20$; $Q(t) = 200 + 6t - 180(1 + \frac{3t}{100})^{-2/3}$
9. $y(0.5) \approx 0.6875$ 10. $-0.8 < t < 1.8$ (see diagram below) 11. $y_2 = t^{-3}$
12. (a) $u = e^{-\frac{t}{2}} \sin t$ (b) E 13. (a) $\frac{1}{s} + \frac{60}{s^4} - \frac{2}{s^2 + 4}$ (b) $\frac{e^{15-5s}}{s-3}$
14. (a) $\frac{1}{3} + \frac{2}{3} e^{-3t}$ (b) $\frac{7}{2} u_3(t) \sin 2(t-3)$ 15. $y = \frac{1}{2} u_3(t) \sin 2(t-3) + \frac{3}{4} \cos 2t + \frac{1}{4}$
16. Let $x_1 = y, x_2 = y'$, then $\begin{cases} x'_1 = x_2 \\ x'_2 = -e^{-t} x_1 + 2t x_2 + 1 \end{cases}$
17. $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ 18. $\vec{x}(t) = e^t \begin{pmatrix} 2 \sin 2t + \cos 2t \\ \cos 2t - \frac{1}{2} \sin 2t \end{pmatrix}$
19. $\vec{x}_p(t) = \frac{e^{2t}}{3} \begin{pmatrix} -2 \\ -8 \end{pmatrix}$ 20. (i) C (ii) A (iii) B

Figure for 10:

