MA 266 Final Practice Problems

1. Find the general solution of these differential equations:

   (a) $y'' - 3y' + 2y = 0$

   (b) $y'' - 6y' + 9y = 0$

   (c) $y'' + 2y' + 2y = 0$

2. Find the general solution of $y''' = 1$

$$y = \boxed{}$$

3. Find the solution of $y' + \frac{3}{t}y = t^2$ such that $y(1) = 2$.

$$y = \boxed{}$$

4. Classify each of the following first order equations as linear, separable, homogenous and/or exact (check all that apply) or none of these:

Differential equation	LINEAR	SEPARABLE	HOMOG	EXACT	NONE
(i) $2x + y + x\,\dfrac{dy}{dx} = 0$					
(ii) $3y\,dx + (2x+1)\,dy = 0$					
(iii) $\dfrac{dy}{dx} = \dfrac{2xy}{x^2+y^2}$					

5. The general solution of $\dfrac{dy}{dx} = \dfrac{2xy + 3x^2}{y^2 - x^2}$ is

A. $\dfrac{y^3}{3} + x^2y + x^3 = C$

B. $\dfrac{y^3}{3} - x^2y - x^3 = C$

C. $2xy + y^2 + 3x^2 = C$

D. $\log(x^2 + y^2) = C$

E. $\dfrac{x^2y + x^3}{\frac{y^3}{3} - x^2y} = C$

6. Give the **FORM** of a particular solution of $y'' - 2y' + y = e^t + \cos t - te^{2t}$.

$y_p = \boxed{}$

7. Find a particular solution of $y'' + y' = 5\cos 2t$.

$y_p(t) = \boxed{}$

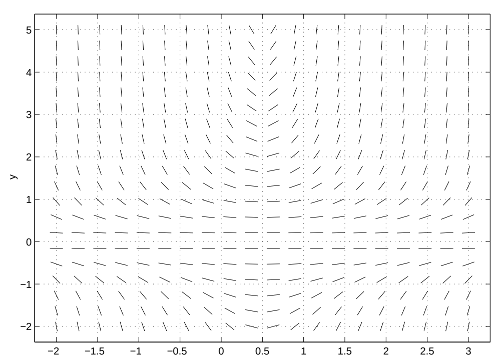
8. At $t = 0$ a tank contains 20 pounds of salt dissolved in 100 gallons of water. A solution that contains 2 pounds of salt per gallon is then pumped into the tank at a rate of 5 gal/min and the well-stirred mixture flows out of the tank at a rate of 2 gal/min. Set up and solve an initial value problem that gives $Q(t)$, the number of pounds of salt at time t.

$Q(t) = \boxed{}$

9. Given the initial value problem $\begin{cases} y' = 2t - y \\ y(0) = 1 \end{cases}$, use the Euler Tangent Line Method (with step size $h = 0.25$) to approximate the true solution at $t = 0.5$.

$y(0.5) \approx$ $\boxed{}$

10. Given the following direction field of the differential equation $\dfrac{dy}{dt} = f(t, y)$, sketch the solution corresponding to the initial value problem with $y(0) = 1$. Approximately where is this solution defined ?



Solution defined approximately for $\boxed{} < t < \boxed{}$

11. Given that $y_1(t) = t$ is a solution of $y'' + \frac{3}{t}\,y' - \frac{3}{t^2}\,y = 0 \quad (t > 0)$, find a second solution $y_2(t)$ which is not a constant multiple of $y_1(t)$.

$y_2(t) = \boxed{}$

12. A spring-mass system with friction is found to satisfy the following initial value data :

$$\begin{cases} 4u'' + 4u' + 5u = 0 \\ u(0) = 0 \\ u'(0) = 1 \end{cases}$$

(a) Solve the initial value problem.

(b) Which graph on the next page best represents the solution ? $\boxed{}$

Figure A:

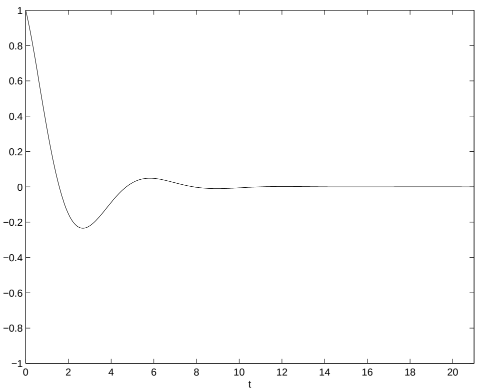


Figure B:

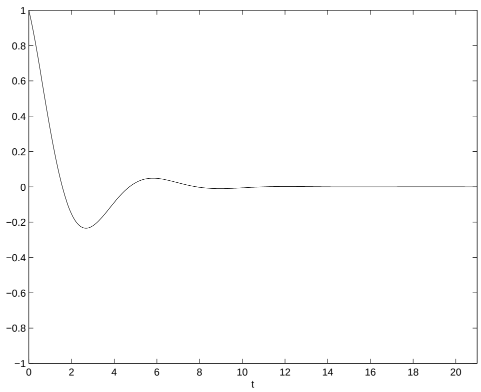


Figure C:

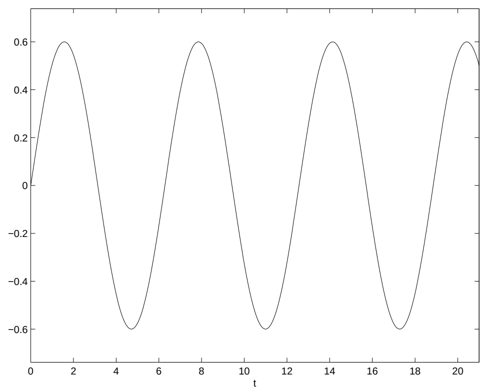


Figure D:

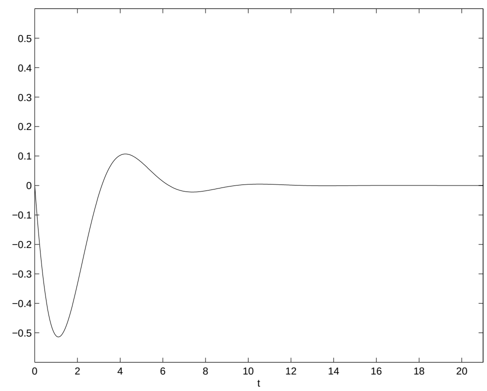


Figure E:

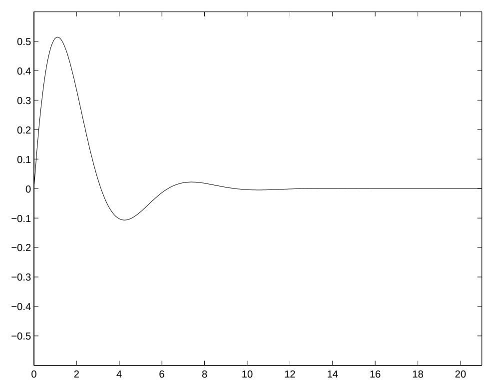
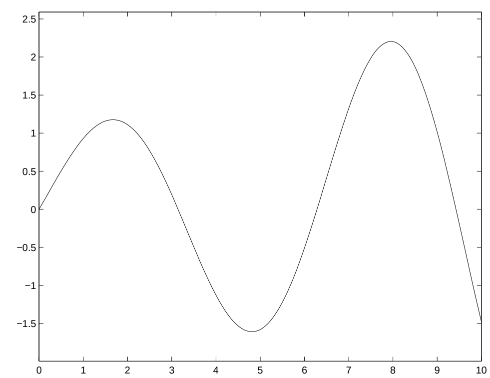


Figure F:



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13. Compute these Laplace transforms:

(a) $\mathcal{L}\{1 + 10t^3 - \sin 2t\} =$ $\boxed{}$

(b) $\mathcal{L}\left\{e^{3t}u_5(t)\right\} =$ $\boxed{}$

where $u_5(t) = \begin{cases} 0\,, & t < 5 \\ 1\,, & t \geq 5 \end{cases}$

14. Find these inverse Laplace transforms:

(a) $\mathcal{L}^{-1}\left\{\dfrac{s+1}{s^2+3s}\right\} =$ $\boxed{}$

(b) $\mathcal{L}^{-1}\left\{\dfrac{7e^{-3s}}{s^2+4}\right\} =$ $\boxed{}$

15. Solve the initial value problem $\begin{cases} y'' + 4y = \delta(t-3) + 1 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$.

$y = \boxed{}$

16. Convert the second order equation $y'' - 2ty' + e^{-t}y = 1$ into a system of first order equations.

17. Find the general solution of this system of equations : $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$$

18. Solve this initial value problem: $\vec{\mathbf{x}}\,'(t) = \begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix} \vec{\mathbf{x}}(t)\,, \qquad \vec{\mathbf{x}}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

$\vec{\mathbf{x}}(t) =$ $\boxed{}$

19. Given that the eigenvalues and corresponding eigenvectors of the 2×2 matrix $\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$ are $r_1 = 3,\ r_2 = -1$ and $\vec{\mathbf{v}}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\vec{\mathbf{v}}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, respectively, find a particular solution of the nonhomogeneous system $\vec{\mathbf{x}}\,'(t) = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \vec{\mathbf{x}}(t) + \begin{pmatrix} 2e^{2t} \\ 0 \end{pmatrix}$.

$\vec{\mathbf{x}}_p(t) =$ $\boxed{}$

20. Match the trajectory that best corresponds to each of the given solutions of a system of differential equations :

(i) $\boxed{}$ $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1\, e^{t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2\, e^{-t} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(ii) $\boxed{}$ $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1\, e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2\, e^{t} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(iii) $\boxed{}$ $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1\, e^{-t} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + C_2\, e^{-t} \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix}$

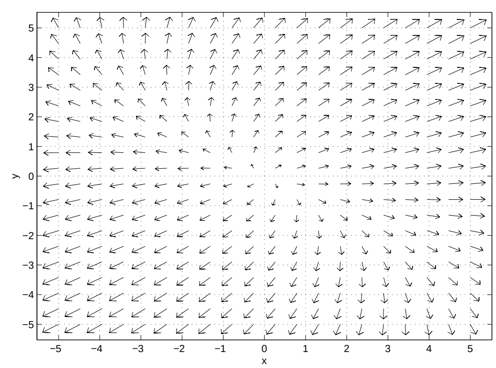

Figure A:

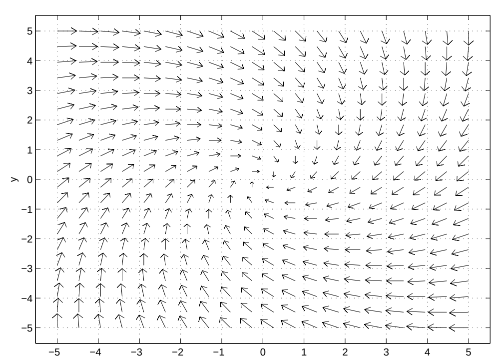

Figure B:

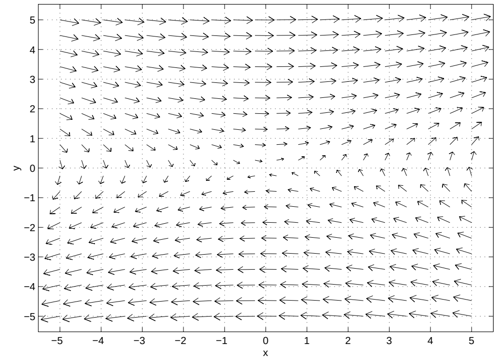

Figure C: