

NAME:

ID.NO:

Problem	Score	Problem	Score
I.		VI.	
II.		VII.	
III.		VIII.	
IV.		IX.	
V.		X.	
		Total	

MA 266, First MidTerm Examination, Wilkerson Sections

75 minutes, Oct. 5, 2000

Do all your work on the question sheets. Calculators are NOT allowed. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper. Each question is worth 10 points.

I. Suppose that $ty' + 3y = t$ for $t > 0$. Solve for $y(t)$. Then find the solution such that $y(1) = 0$. Finally, for this y , calculate $y(e)$.

II. Give the general solution of the differential equation

$$(x^2 + y)dy + (2xy + 1)dx = 0.$$

in a box $2 < x < 5, 1 < y < 3$. If $y(3) = 2$, give the solution with this initial value.

III. Suppose that a tank initially contains 35 gallons of a salt water solution with a concentration of 10 lbs/gallon. A salt water solution with a concentration of 0.3 lb/gallon is poured into the tank at a rate of 5 gallons/min and the well stirred solution exits the tank at the rate of 4.9 gallons/min. Suppose in addition that 0.1 gallon of pure water evaporates a minute. First write a first order differential equation that predicts the quantity of salt in the tank at time t . Solve this differential equation and calculate the concentration of the solution in the tank after 10 minutes.

IV. A small meteor falls into the Wabash river at midnight. The first diver reaches it a 6AM that morning and his hand thermometer indicates that the temperature of the meteor is 180 degrees F. By the time the meteor is hauled out of the river at 8AM, the temperature has dropped to 150 degrees. Use Newton's law of cooling to estimate the temperature of the meteor when it first hit the Wabash. Assume that the Wabash has a constant temperature of 68 degrees F.

V. Convert the differential equation $y'' - 2y'/t = 0$ into a first order linear equation by the substitution $v = y'$, then give the new equation, and solve for $v(t)$.

VI. Give two independent solutions $\{y_1, y_2\}$ to

$$y'' + 5y = 0$$

and demonstrate their independence by calculating the Wronskian of $y_1 = y_2$ at $t = 0$.

VII. Pick two linearly independent solutions $\{y_1, y_2\}$ of the second order constant coefficient linear homogeneous equation

$$y'' - 16y = 0$$

and compute the Wronskian $W(y_1, y_2)$. Finally, choose a solution $y(t)$ such that $y(0) = 5$ and $y'(0) = 7$.

VIII: Give the general solution to $y'' + 6y' + 10y = 0$ and the only solution for which $y(0) = 0$ and $y'(0) = 1$. What is the limit of this $y(t)$ as $t \rightarrow \infty$

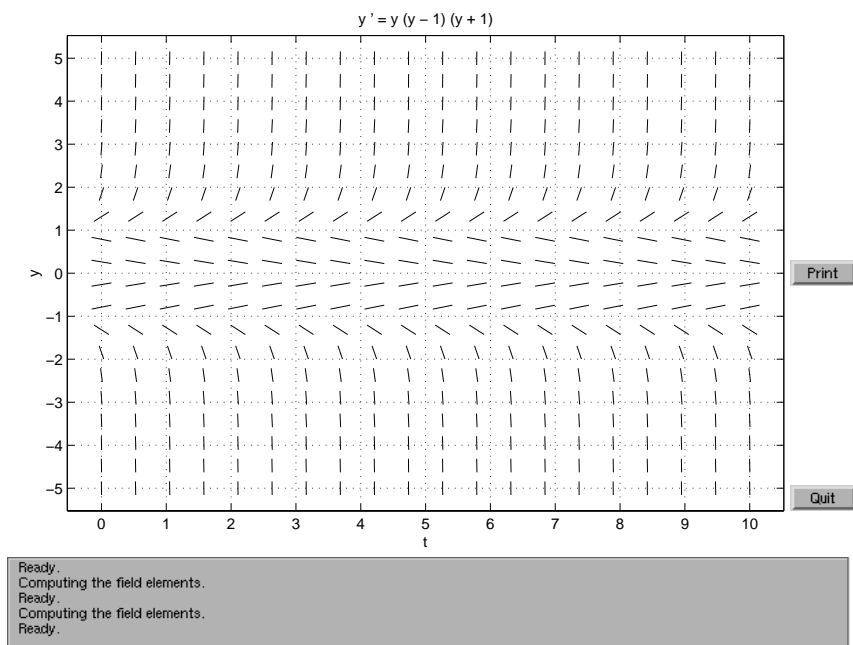
IX. Make these conversions between complex exponentials and complex numbers :

a) $e^{-3i\pi/4} = \text{????}1 + \text{????}i$

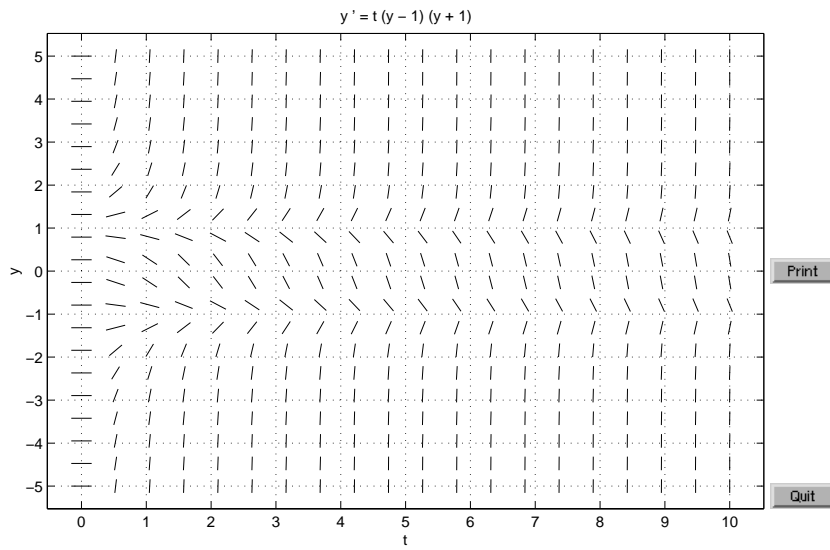
b) $3 + 3i = \exp(\text{????})$

X. For the ODE $y' = y(y^2 - 1)$, choose the graph below that best represents its direction fields. Using this graph and the equation, a) plot the solutions for the initial conditions for $\{y(1) = 2, y(1) = 0.5, y(1) = 0, y(1) = -0.4, y(1) = -0.5\}$ (5 trajectories in all), b) based on the graph, estimate the values of $t > 0$ for which the solution passing through $y(1) = 2$ is valid, c) based on the graph, estimate the limit of $y(t)$ as $t \rightarrow \infty$ for the solution passing through $y(1) = 0.5$.

Graph 1.



Graph 2.



Ready.
Computing the field elements.
Ready.
Computing the field elements.
Ready.