Math 366 Review Problems, First set, Fall 01.

1. If $y = y(x)$ satisfies $y' = \dfrac{2x(y-1)}{x^2+3}$, $y(1) = -3$ then $y(\sqrt{2}) =$

2. A hot object is placed in a room whose temperature is 72^oF. After two minutes the temperature of the object is 172^o and after 4 minutes the temperature is 122^oF. The initial temperature of the object is:

3. The matrix $A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 4 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ has 3 as an eigenvalue. What is the dimension of the eigenspace corresponding to the eigenvalue 3?

4. The general solution of $(D^4 - 1)y = 0$ is

5. If the system $x' = Ax$ has the fundamental matrix $\Psi = \begin{bmatrix} e^t & e^{2t} \\ -2e^t & 2e^{2t} \end{bmatrix}$, then the solution to $\mathbf{x}' = A\mathbf{x},\ \mathbf{x}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is

6. The solution to $y'' + 4y' + 6y = 0$ can be found by solving the linear system $\mathbf{x}' = A\mathbf{x}$ where $A =$

7. Use the definition of $\sin x$ and $\cos x$ in terms of complex exponentials, to show that $\sin 2x = 2\sin x \cos x$.

8. Find the general solution to the system

$$\mathbf{x}' = A\mathbf{x}, \qquad A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -3 & -1 \\ -1 & 1 & -1 \end{bmatrix}$$

(hint: A is a defective matrix).

9. (a) Find the general solution $x(t)$ to the differential equation

$$\ddot{x} + 3\dot{x} + 2x = 10\sin t,$$

and express the steady–state behavior in the form $A\cos(t-\phi)$.

(b) Do solutions to this equation form a vector space? Why or why not?

10. Solve by variation of parameters

$$y'' - 6y' + 13y = 4e^{3x}\sec^2 2x \qquad (0 \le x \le \pi/4).$$