Math 366 Homework/Review

Due Nov. 12

1. Solve for the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

2. Given that the eigenvalues of the matrix $A = \begin{pmatrix} 4 & -6 \\ 3 & -5 \end{pmatrix}$ are 1 and -2, find a (nonzero) eigenvector for each eigenvalue.

3. Convert the second order equation $y'' - y' + 4y = 0$ into a system of first order equations in $x_1(t)$ and $x_2(t)$, where $x_1(t) = y(t)$ and $x_2(t) =??$.

4. For the second order inhomogeneous D.E.

$y'' - 2y' + y = e^t + cos(t) - te^2$, give the general form of the solution. Find the solution with $y(0) = 0$ and $y'(0) = 0$.

5.(a) Given that the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has eigenvalues 1 and -1 with the eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ respectively, give the solution to the initial value problem $\vec{x}(t)' = A\vec{x}(t)$ and $\vec{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(b). Give a particular solution to $\vec{x}(t)' = A\vec{x}(t) + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Here A is the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(c) Find the matrix e^{At} by your choice of methods. Show your work.

(a) $\vec{x}(t) =$

(b) $\vec{x}_p(t) =$

(c) $e^{At} =$

6. For the problem

$$\vec{x}(t)' = \begin{pmatrix} 2 & 5 \\ 5 & 2 \end{pmatrix} \vec{x}(t) + \begin{pmatrix} e^t \\ 0 \end{pmatrix},$$

(a) find the eigenvalues of A.

(b) find a (nonzero) eigenvector for each eigenvalue. (c) give the general solution to the homogeneous problem. (d) give a particular solution of the form $e^t\vec{v}$ to the inhomogeneous problem, where $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ is a constant vector that you must find.

(a)	(b)
(c)	(d)

7. Do problem 5 above using the variation of parameter method.

8. Find $exp(At)$ for each of these matrices:

1. $A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$

2. $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

3. $A = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$