NAME: ____________________

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MA 366, Fall 2001 Second Examination Wilkerson Section Only.

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Read each problem carefully. It may NOT be asking for as much information as you first think. Write your answer in the box provided.

1.(20 points) Use your choice of methods to give a solution

$$y'' + 2y' + y = e^{-t}$$

with $y(0) = 1$ and $y'(0) = 0$.

2.(10 points) Solve for the eigenvalues of the matrix $A = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$

3.(10 points) Given that the eigenvalues of the matrix $A = \begin{pmatrix} 3 & -6 \\ 1 & -2 \end{pmatrix}$ are 0 and 1, find a (nonzero) eigenvector for each eigenvalue.

4.(20 points) For the problem

$$\vec{x}(t)' = \begin{pmatrix} 2 & 0 \\ 5 & 1 \end{pmatrix} \vec{x}(t) + \begin{pmatrix} e^{-t} \\ 1 \end{pmatrix},$$

the eigenvalues A are 1 and 2. (a) find a (nonzero) eigenvector for each eigenvalue.

(b) give the general solution to the homogeneous problem.

(c) find a particular solution of the form $e^{-t}\vec{v} + \vec{u}$ to the inhomogeneous problem, where $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ is a constant vector that you must find, and $\vec{u}$ is another constant vector that you must find. (you can do this part without having done (b))

(d) Give the solution corresponding to initial value $x_1(0) = 1,\ x_2(0) = 1$

vector 1	vector 2

homogeneous solution

particular solution

Initial value solution

5. Exponential functions.

1. Write the power series for $e^t = exp(t)$. (5 points)

2. The matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has the property that $A^2 = Id$. Use this and the power series above to write the first 5 non-zero terms of $e^{At} = \exp(At)$. Give each component of the matrix. (7 points)

3. Use the eigenvalue-eigenvector method to solve the system

$$\vec{x}(t)' = A\vec{x}(t)$$

where A is the matrix above, for the solution with $\vec{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (8 points).

6. It is known that for the matrix $B = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$,

$$\exp(Bt) = e^{Bt} = e^{\lambda t} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}.$$

1. Suppose that $\lambda = 2$. Use the formula for e^{Bt} and the variation of parameters method to give an integral formula for the solution of the DE system

$$\vec{x}(t) = B\vec{x}(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(10 points)

2. Either complete the computation of the solution above for $\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, or solve using any other method. (10 points).