NAME:

ID.NO:

Problem	Score	Problem	Score
I.(12)		II.(12)	
III.(15)		IV.(14)	
V.(15)		VI.(14)	
VII.(18)			
		Total	

MA 266, First MidTerm Examination, Wilkerson Section 8:30AM MWF

50 minutes, Feb. 7, 2001

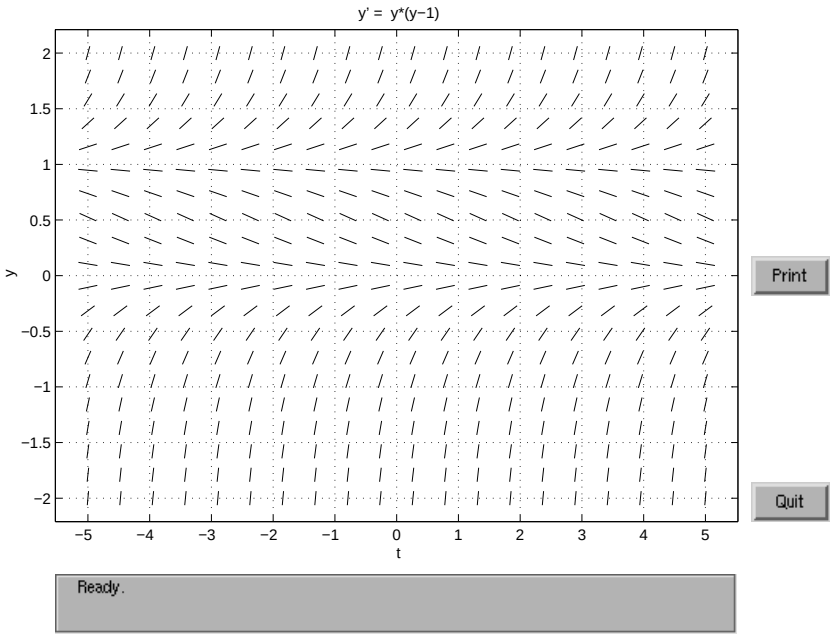
I. Suppose that $ty' + 2y = 5t$ for $t > 0$. Solve for $y(t)$. Then find the solution such that $y(1) = 1$. Finally, for this y, calculate $y(2)$. (12 pts)

II. Suppose that a tank initially contains 100 gallons of a salt water solution with a concentration of 1 lbs/gallon. A salt water solution with a concentration of 2.0 lb/gallon is poured into the tank at a rate of 5 gallons/min and the well stirred solution exits the tank at the rate of 4.9 gallons/min. Suppose in addition that 0.1 gallon of pure water evaporates a minute. SETUP the first order differential equation and initial conditions that predict the quantity of salt in the tank at time t. Do NOT SOLVE this differential equation . HINT: draw and label a picture of the problem. (12 pts)

III. A small meteor falls into the Wabash river at midnight. The first diver reaches it at 6AM that morning and his hand thermometer indicates that the temperature of the meteor is 180 degrees F. By the time the meteor is hauled out of the river at 8AM, the temperature has dropped to 150 degrees. Use Newton's law of cooling to estimate the temperature of the meteor when it first hit the Wabash. Assume that the Wabash has a constant temperature of 60 degrees F. (15 pts)

IV.Convert the differential equation $y'' - 3y'/t = 0$ into a first order linear equation by the substituition $v = y'$, then give the new equation, and solve for $v(t)$ and $y(t)$. Next assume that $y(1) = 0$ and $y'(1) = 1$ and give a solution matching these initial conditions. (14 pts)

V. For the ODE $y' = y(y-1)$, first use the direction field below to sketch the solution for each of the initial points (a) $y(0) = -1$ (b) $y(0) = 0$, (c) $y(0) = 1/2$, (d) $y(0) = 1$, and (e) $y(0) = 2$. Second, for the initial value $y(0) = 1/2$, estimate the largest interval in t which contains $t = 0$ where the solution seems to be defined. (15 pts)



VI. Existence and uniqueness. (14 pts)

(a) Give the largest t-interval in which the ODE $ty' + y/(t-1) = t^2, \quad y(1/2) = 2$ has a unique solution. Justify your answer.

(b) Give two different solutions to $y' = y^{1/3}, \quad y(0) = 0$. Explain why this does not contradict the

uniqueness theorem for first order equations.

VII. Solve each of these first order equations (18 pts)

(a) $y' = 5y, \quad y(0) = 3.$

(b) $y' = y(y-2), \quad y(0) = 1$

(c) $y' = xy/(x^2+y^2), \quad y(1) = 1$