

MA 366: Introduction to Differential Equations

Fall 2001, Test One

Instructor: Yip

- This test booklet has FIVE QUESTIONS, totaling 50 points for the whole test. You have 60 minutes to do this test. **Plan your time well. Read the questions carefully.**
- This test is closed book and closed notes.
- (Any kind of) calculator is allowed. But you should **not** use it whenever it is possible (from the point of view of this class), i.e. your answers should be as **analytical** as possible.
- In order to get full credits, you need to give **correct** and **simplified** answers and explain in a **comprehensible way** how you arrive at them.
- You can use both sides of the papers to write your answers. But please indicate so if you do.

Name: _____ (Major: _____)

| <u>Question</u> | <u>Score</u> |
|-----------------|--------------|
| 1.(10 pts) | _____ |
| 2.(10 pts) | _____ |
| 3.(10 pts) | _____ |
| 4.(10 pts) | _____ |
| 5.(10 pts) | _____ |
| Total (50 pts) | _____ |

1. Find the general solution to the following differential equation:

$$y' + \frac{1}{x}y = \frac{1}{y^2}, \quad y(1) = 1.$$

(Hint: Multiply by y^2 and let $u = y^3$.)

2. The growth law for the population of a (strange kind of) virus equals two times the square of the population. Suppose initially that the population is 5(units).

- (a) Find a formula for the population at time t .
(b) Will the population explode? If so, when does this happen?

3. Find the solution for the the following initial value problem:

$$y'' - y' - 6y = 2 + e^{-2x}, \quad y(0) = 1, \quad y'(0) = 0$$

Determine also the value of $\lim_{x \rightarrow +\infty} y(x)$.

4. Consider the following three examples of differential equations:

$$y' = y^2 + 2y - 15$$

$$y' = y^4 - 1$$

$$y' = (e^{2y} - 1)(y^2 - 4)$$

For each of the above,

- (a) Find all their equilibrium points and determine their stability properties.
(b) Sketch the solution $y(x)$ starting from $y(0) = 2$. Pay attention to the behavior when x is *large*.
5. A 100-gallon tank is initially half full of pure water. Three gallons per minute of salt solution with salt concentration being one pound per gallon is flown into the tank. On the other hand, one gallon per minute of the mixed solution flows out of the tank. In addition, one gallon of water evaporates from the tank.

How much salt is in the tank when it begins to overflow?