

NAME _____

Id. No, _____

First Examination, Math 453, Fall 03 8:30MWF Wilkerson Section.

October 7, 2003, 7:30PM-9:00PM, University 103

No notes, books, calculators, tape players, earphones. Show all work. Use back of pages for scratch. Ask if you have questions on how much work needs to be shown, or what can be assumed.

Problem	Score
Longer	
1.(10)	
2.(10)	
3.(10)	
4.(10)	
5.(10)	
Shorter	
6.(25)	
7.(25)	
Total(100)	

1. 10 points

- (a) List the eight eighth roots of unity (evaluate trig functions !).
- (b) For each root from (a), list its order.
- (c) List the primitive eighth roots of unity.

2. 10 points

Use proof by induction to establish the formula

$$\sum_{k=1}^n (2k - 1) = n^2$$

State "anchor" step.

State the induction hypothesis.

Show that if the statement is true for n then it is true for $n + 1$.

3. 10 points : Cubic equation:

For the cubic equation

$$y^3 + py + q = 0$$

the substitution $y = z - p/(3z)$ leads to the formula

$$z^3 = (-q \pm \sqrt{q^2 + 4p^3/27})/2$$

Carry out this for

$$x^3 + 9x - 6 = 0.$$

(solve for z and use this to find the roots of the original equation).

4. 10 points: Application of binomial theorem.

The binomial theorem states that

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k.$$

(a) Use this to evaluate

$$\sum_{k=0}^n \binom{n}{k} 2^k$$

for general $n > 0$. Then check it for $n = 4$.

(b) Compute the sum

$$\binom{n}{1} + 2\binom{n}{2} + \dots + n\binom{n}{n}$$

for general $n > 1$. Check your answer for $n = 4$.

5. 10 points: Jack and Jill live with their mother at the top of a steep hill. The well is in the valley below. They have a large water bucket with volume 11 quarts and a smaller one of volume 3 quarts. The water barrel by their house is fixed, but has a faucet for easy drainage. One morning, their mother discovers that the barrel is empty. Since it was Purdue homecoming weekend, she tells the kids to just put exactly one quart of water into the barrel before they leave for the game. How can they accomplish this? Is there more than one way to do it? What if the larger bucket were 12 quarts instead of 11? Hint: Euclidean Algorithm and GCD's.

6. 25 points: Short answer.

(a) Use the E.Alg. to find the multiplicative inverse of 5 in $Z/17$?

(b) Write Pascal's triangle up to and including the expansion of $(a + b)^5$.

(c) List the integers $0 \leq a < 11$ such that $a = b^2$ in $Z/11$ for some integer b .

(d) Write out the multiplication table for $Z/9$.

(e) Find the solutions to

$$x^{17} + x^5 + x^3 = 0$$

over $Z/5$.

7. 25 points: Short answer.

(a) If $z = 3 - i(\sqrt{3})$, give $|z|$, the modulus.

(b) If $z = 2 - 2i$, what is $\arg(z)$?

(c) Express

$$1/(2 - \sqrt{3}i)$$

in the form $a + bi$, where a and b are real numbers.

(d) Find the roots of the equation $x^2 = 1$ in

i) $Z/5$ and

ii) $Z/8$.

(e) Calculate the Euler *phi*-function $\phi(264)$. Hint: do a factorization of 264.