MA453 HW #10 Solutions.

§9.3 #6 When the symbols a, b, c, d, e of Table 9.9 are replaced by $\alpha, \gamma, \delta, \varepsilon, \beta$ respectively, Table 9.11 is obtained.

$\therefore$ groups whose multiplication tables are in Table 9.9 and 9.11 are isomorphic to each other.

#9. (i) $f(a) = f(b) \Leftrightarrow 3a = 3b$

then $a \equiv 67 \cdot 3a = 67 \cdot 3b \equiv b$ in $\mathbb{Z}_{100}$

$\therefore a = b$ in $\mathbb{Z}_{100}$ $\quad \therefore f$ is 1-1.

(ii) Since $\mathbb{Z}_{100}$ is finite group, by (i), f is onto.

(iii) $f(a+b) = 3(a+b) = 3a+3b = f(a)+f(b)$

#12. Since 17 is prime, $\forall x \in \mathbb{Z}_{17}$, $\exists x^{-1} \in \mathbb{Z}_{17}$.

$f(f(x)) = f(x^{-1}) = (x^{-1})^{-1} = x$ $\quad \therefore f$ is 1-1 & onto

$f(xy) = (xy)^{-1} = y^{-1}x^{-1} \underset{\uparrow}{=} x^{-1}y^{-1} = f(x)f(y)$

$\because \mathbb{Z}_{17}$ is abelian

§9.4 #28. $|S_5| = 5!$

Since $50 \nmid 120$, by Lagrange, S_5 doesn't contain a subgroup of order 50.

#2. $n=1$. $|S_1| = 1$ $\quad \therefore \exists$ only one subgp S_1

$n=2$ $\quad |S_2| = 2! = 2$ $\quad \therefore S_2$ & $\{Id\}$

$n=3$ $\quad S_3$, $\{Id, (1\ 2\ 3), (1\ 3\ 2)\}$,

$\{Id, (1\ 2)\}$, $\{Id, (1\ 3)\}$, $\{Id, (2\ 3)\}$

$\{Id\}$