§6.1 p105. #2.

$$
\begin{array}{r|l}
 & x^4+2x^3+x^2+2x+2 \\
\hline
x^3+x^2+1 & x^7 \qquad\qquad +x^4 \qquad\qquad +x+1 \\
 & \underline{x^7+x^6 \qquad +x^4} \\
 & 2x^6 \\
 & \underline{2x^6+2x^5 \qquad +2x^3} \\
 & x^5 \qquad +x^3 \\
 & \underline{x^5+x^4 \qquad +x^2} \\
 & 2x^4+x^3+2x^2 \\
 & \underline{2x^4+2x^3 \qquad +2x} \\
 & 2x^3+2x^2+2x \\
 & \underline{2x^3+2x^2 \qquad +2} \\
 & 2x+2
\end{array}
$$

$\therefore$ quotient: $x^4+2x^3+x^2+2x+2$, remainder : $2x+2$

§6.2 p111. #12.

$$P(x) = x^{25}-2x^4+3x^3-4x^2+5x-1 = (x-1)Q(x)+R(x)$$

$$x=1 \Rightarrow \quad P(1) = 1-2+3-4+5-1 = 0+R \qquad \therefore R=2$$

$\therefore$ remainder $=2$

§6.3 p115. #1.

$$x^6+x^5+x^4+x^3+1=(x+1)(x^5+x+1)+(x^4+x^3+x^2)$$

$$x^5+x+1=(x+1)(x^4+x^3+x^2)+(x^2+x+1)$$

$$x^4+x^3+x^2=x^2(x^2+x+1)$$

$\therefore$ greatest common divisor : x^2+x+1

p116 #4.

$$x^7+x^6+x^5+2x^3+x^2+2x+1=x(x^6+x^5+x^4+2)+(2x^3+x^2+1)$$

$$x^6+x^5+x^4+2=(2x^3+x^2+2)(2x^3+x^2+1)$$

$\therefore$ greatest common divisor : $2x^3+x^2+1$

§6.3 p116 #11.

Suppose there exist polynomials $A(x)$ and $B(x)$ such that

$$A(x)(x^2-5x+6)+B(x)(x^2+x-6)=x^2+1.$$

Then degree of $A(x)(x^2-5x+6)+B(x)(x^2+x-6)$ = degree of x^2+1

$\therefore$ $\max\{$degree of $A(x)(x^2-5x+6)$, degree of $B(x)(x^2-5x+6)\}=2.$

Since degree of $A(x)(x^2-5x+6)$ = degree of $A(x)+2$ and

degree of $B(x)(x^2+x-6)$ = degree of $B(x)+2$,

$2+\max\{$degree of $A(x)$, degree of $B(x)\}=2.$

$\therefore$ $\max\{$degree of $A(x)$, degree of $B(x)\}=0.$

$\therefore$ degree of $A(x)$ = degree of $B(x)=0$

$\therefore$ $A(x)$ and $B(x)$ are constants.

Then let $A(x)=a$ & $B(x)=b$.

And $a(x^2-5x+6)+b(x^2+x-6)=(a+b)x^2+(-5a+b)x+(6a-6b)=x^2+1.$

$$\therefore \begin{cases} a+b=1 & \cdots (1) \\ -5a+b=0 & \cdots (2) \\ 6a-6b=1 & \cdots (3) \end{cases}$$

(1)−(2) $6a=1$ $\therefore a=1/6$

plug $a=1/6$ in (1), $b=1-1/6=5/6.$

But $6a-6b=1-5=-4\neq 1$ in (3).

$\therefore$ there is no (a,b) such that satisfies (1), (2) and (3).

$\therefore$ No