**Group Actions, Toric Varieties and Birational Geometry.**

This page contains information for the course MA696W at Purdue University.
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**Summary:**
Prerequisites. Basic knowledge about algebraic geometry (like R.Hartshorne 'Algebraic Geometry' Chapter I or similar).

The purpose of this course is to give a survey on various techniques used in birational geometry and its interactions with invariant theory and toric geometry. We will introduce algebraic group actions, good and geometric quotients, $\mathbb{C}^*$-actions, Białynicki-Birula decomposition, reductive groups, geometric invariant theory, Luna's slice theorem, birational cobordisms (techniques inspired by topological cobordisms), and elements of Mori theory. In the course we introduce and briefly discuss the theory of toric varieties as the illustration of the above mentioned techniques with particular emphasis on Mori theory, Morelli cobordisms and $\mathbb{C}^*$ -actions and the theory of valuations.

One of the main goals will be the sketch of a proof of the Weak Factorization Theorem which states that any birational map between smooth projective varieties is a composition of blow-ups and blow-downs along smooth centers.

The seminar should deal with the following:

- We prove the classical Białynicki-Birula decomposition theorem (for $\mathbb{C}^*$-actions)
- We introduce birational cobordisms, and GIT for torus actions and show the relation with birational factorization
- Introduce the reductive groups and the classical notion of categorical, good and geometric quotients $X//G$ for $G$ reductive.
- We extend the results on GIT for the reductive group actions.
- We prove the linear reductivity of some classical groups in characteristic zero, and give the proof of Hilbert’s 14th problem for reductive groups.
- We discuss the classical Luna's etale and Luna's slice Lemma for Torus and Reductive group actions.
- We show the Hilbert-Mumford criterion for stability for the reductive groups.
- We introduce toric varieties and illustrate the above concepts in the toric setting

The focus of this course is to give an intuition about the interplay of different areas of algebraic geometry.

**Main texts:**

- M. Brion. *Introduction to actions of algebraic groups.*
- M. Bukstedt. *Notes on Invariant group Theory*
- J.Wlodarczyk *Algebraic Morse Theory and Factorization of Birational Maps.*
Additional texts:

Bialynicki-Birula  Some theorems on group actions.
Igor Dolgachev  Lectures on Invariant Theory
Jerzy Konarski  The B-B decomposition via Sumihiro Theorem
Tadao Oda  Convex bodies and Toric Varieties
Kenji Matsuki  Introduction to Mori Theory
V.L. Popov and E. B. Vinberg  Invariant Theory in Algebraic Geometry 4.
T.A. Springer  Linear Algebraic Groups

Tentative contents of the course.

Final exam project information.