Math 265 Linear Algebra, Spring 2002

First Midterm

Please be neat and show all work. Write each answer in the provided box. Use the back of the sheets and the last 3 pages for extra scratch space. Return this entire booklet to your instructor. **No books. No notes. No calculators.**

0.(2 points)

Student Name (print):	Student ID:

Problem #	Max pts.	Earned points
1	**6**	
2	**6**	
3	**3**	
4	**4**	
5	**6**	
6	**4**	
7	**4**	
8	**4**	
9	**4**	
10	**1**	
Section I	**42**	

11	**6**	
12	**4**	
13	**6**	
14	**6**	
15	**2**	
Section II	**24**	
16	**8**	
17	**8**	
18	**8**	
19	**8**	
Section III	**32**	
TOTAL	**100**	

SECTION I: SHORT PROBLEMS

Show all your work. Write your answer **clearly** in the provided box. **No partial credit on this part**.

1.(6 points) Let $A = \begin{bmatrix} 1 & 6 & 1 & 2 & 0 \\ 3 & 18 & 2 & 5 & -1 \\ -1 & -6 & 5 & 4 & 6 \end{bmatrix}$ MATLAB gives that

$$\text{rref}(A) = \begin{bmatrix} 1 & 6 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \text{rref}(A^T) = \begin{bmatrix} 1 & 0 & 17 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) **(2 points)** What is the dimension of the null space of A?

(b) **(2 points)** Find a basis for the **column** space of A.

(c) **(2 points)** True or False. The vector $\begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$ in the **column** space of A.

2.(6 points) Consider the system

$$\begin{aligned} x_1 + x_2 &= 2 \\ x_1 + (a^2 - 5)x_3 &= a \\ x_2 + 4x_3 &= 1. \end{aligned}$$

Find all values of a for which the system has

(a) exactly one solution. (b) no solution. (c) infinitely many solutions.

(a)

(b)

(c)

3.(3 points) For what values of s is the vector $\begin{bmatrix} 0 \\ 1 \\ s \end{bmatrix}$ in span $\left\{ \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 1 \end{bmatrix} \right\}$?

4.(4 points) Let $W = \text{span}\left\{ \begin{bmatrix} 1 \\ -4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 12 \\ -9 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ -8 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \\ -1 \end{bmatrix} \right\}$. Find a basis for W.

5.(6 points) Give a system

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = & 1, \\ x_1 + 2x_2 + 3x_3 + 4x_4 = & 0, \\ 2x_1 + 3x_2 + 4x_3 + 5x_4 = & 0. \end{cases}$$

(a) **(3 points)** If we write it in matrix form; i.e., $A \cdot \mathbf{x} = \mathbf{b}$. What are A, $\mathbf{x}$ and $\mathbf{b}$?

$A =$

$\mathbf{x} =$

$\mathbf{b} =$

(b) **(3 points)** What is the nullity of A?

6.(4 points) Find the inverse matrix of $A = \begin{bmatrix} 4 & 7 \\ 5 & 9 \end{bmatrix}$ if A has one.

7.(4 points) Let L be a mapping from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined as follows: $L\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 + a_2 \\ 2a_2 \\ a_1 - a_2 \end{bmatrix}$.

(a) **(2 points)** What is $L\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)$?

(b) **(2 points)** True or false. L is onto $\mathbb{R}^3$.

8.(4 points) Find the dimension of the row space of $A = \begin{bmatrix} 1 & -1 & 3 & 4 \\ -2 & 2 & -5 & -7 \\ -6 & 6 & -1 & -7 \end{bmatrix}$.

9.(4 points) Let $A = \begin{bmatrix} 7 & 5 \\ 10 & 7 \end{bmatrix}$. Write A^{-1} as a product of elementary matrices.

$A^{-1} =$

10.(1 points) Given two matrices $A = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix}$. Compute $(AB)^T$.

$(AB)^T =$

SECTION II: MULTIPLE CHOICE–NO PARTIAL CREDIT

For Problems 11 through 15, circle **only one** (the correct) answer for each part. **No Partial credit.**

11.(6 points) Determine whether the following are true or false:

(a) **(2 points)** If $A_{m\times n}$ and $B_{n\times p}$ are two matrices, then $(AB)^T = A^T B^T$. True False

(b) **(2 points)** If $A_{n\times n}$ and $B_{n\times n}$ are two matrices, then $(AB)^{-1} = B^{-1}A^{-1}$. True False

(c) **(2 points)** If A is non-singular, then $A\mathbf{x} = \mathbf{b}$ has nontrivial solution. True False

12.(4 points) Determine if each of the following sets of the vectors forms a basis for $\mathbb{R}^4$?

(a) **(1 points)** $\left\{ \begin{bmatrix} 7 \\ 2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 6 \\ -9 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 7 \\ 1 \end{bmatrix} \right\}$. True False

(b) **(1 points)** $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \\ 11 \end{bmatrix} \right\}$ True False

(c) **(1 points)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} \right\}$ True False

(d) **(1 points)** $\left\{ \begin{bmatrix} 5 \\ 7 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -9 \\ 5 \\ 8 \\ 10 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -3 \\ 1 \end{bmatrix} \right\}$. True False

13.(6 points) Determine if each of the following sets of the vectors is a subspace for $\mathbb{R}^4$.

(a) **(2 points)** $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, \; a + b = c + d, \; a, b, c, d \in \mathbb{R} \right\}$. True False

(b) **(2 points)** $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, \; a = c + d - 1, \; a, b, c, d \in \mathbb{R} \right\}$. True False

(c) **(2 points)** $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, \; c + d \geq -1, \; a, b, c, d \in \mathbb{R} \right\}$. True False

14.(6 points) Determine if each of the following sets of the vectors **spans** the null space of

$$A = \begin{bmatrix} 1 & -1 & 3 & -2 \\ -1 & 0 & -4 & 3 \\ 1 & -2 & 2 & -1 \end{bmatrix}.$$

(a) **(2 points)** $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$ True False

(b) **(2 points)** $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}.$ True False

(c) **(2 points)** $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}.$ True False

15.(2 points) M_{66} is isomorphic to M_{49}. True False

SECTION III: MULTI-STEP PROBLEMS – PARTIAL CREDIT

Show all work (**no work - no credit!**) and display computing steps. Write clearly.

16.(8 points) Given a system

$$\begin{cases} x_1 - x_2 + x_3 - 2x_5 = & -1, \\ 5x_1 - 5x_2 + 6x_3 + x_4 - 9x_5 = & -7, \\ 2x_1 - 2x_2 + 3x_3 + x_4 - 3x_5 = & -4, \\ x_1 - x_2 + 2x_3 + x_4 - x_5 = & -3. \end{cases}$$

(a) **(1 points)** What is the augmented matrix?

(b) **(7 points)** Find the general solution.

17.(8 points) Given a matrix

$$A = \begin{bmatrix} 2 & -6 & 4 & 6 & -2 \\ -1 & 3 & -3 & -4 & 2 \\ -2 & 6 & -3 & -5 & 1 \\ 1 & -3 & -1 & 0 & 2 \end{bmatrix}.$$

(a) **(3 points)** What is the rank of A?

(b) **(3 points)** Find a basis for the null space of A.

(c) **(2 points)** What is the rank of A^T?

18.(8 points) Let $S = \{t^2 - t - 1, -t^2 + 2t - 1, t^2 + t - 5\} \subset P_2$.

(a) **(2 points)** True or false. S is linearly independent. True False

(b) **(4 points)** Find a basis for $\operatorname{span} S$.

(c) **(2 points)** Does S span the whole P_2? True False

19.(8 points) Let V be $\mathbb{R}_3$ and an ordered set

$$S = \{\begin{bmatrix}1 & 2 & 3\end{bmatrix},\ \begin{bmatrix}2 & 4 & 5\end{bmatrix},\ \begin{bmatrix}4 & 7 & 12\end{bmatrix}\}.$$

(a) **(2 points)** Verify the set S is a basis for $\mathbb{R}_3$.

(b) **(4 points)** Given a vector $\mathbf{v} = \begin{bmatrix}1 & 1 & 2\end{bmatrix}$, compute $\begin{bmatrix}\mathbf{v}\end{bmatrix}_S$.

(c) **(2 points)** What is the row rank of the matrix $\begin{bmatrix}1 & 2 & 4\\ 2 & 4 & 7\\ 3 & 5 & 12\end{bmatrix}$?

Use this sheet for extra scratch space. Do not remove it from the booklet.

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