Math 265 Linear Algebra, Spring 2002

Practice Exam for Second Midterm

Please be neat and show all work. Write each answer in the provided box. Return this entire booklet to your instructor. **No books. No notes. No calculators.**

Student Name (print):	Student ID:

Circle your class: Div 7, Sec 3 (9:00) Div 8, Sec 2 (10:30)

Problem #	Max pts.	Earned points
1	**4**	
2	**4**	
3	**4**	
4	**4**	
5	**4**	
6	**4**	
7	**4**	
8	**4**	
9	**6**	
10	**6**	
Section I	**44**	

11	**6**	
12	**4**	
13	**4**	
14	**6**	
15	**4**	
Section II	**24**	
16	**10**	
17	**8**	
18	**6**	
19	**8**	
Section III	**32**	
TOTAL	**100**	

SECTION I: SHORT PROBLEMS

Show all your work. Write your answer **clearly** in the provided box. **No partial credit on this part**.

1.(4 points) Let $\mathbf{v_1} = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$. Let θ be the angle between $\mathbf{v_1}$ and $\mathbf{v_2}$. Find $\cos\theta$.

$\theta =$

2.(4 points) Let $A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{bmatrix}$. Computer $\det(A)$.

$|A| =$

3.(4 points) Let W be the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} , \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} , \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. Find an orthogonal basis of the space that is orthogonal to W; i.e., $W^{\perp}$.

4.(4 points) A linear transformation $L\colon \mathbb{R}^5 \to \mathbb{R}^3$ defined by $L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}\right) = \begin{bmatrix} a_1 - 2a_2 + 5a_3 - 3a_5 \\ 5a_1 + 2a_2 - 6a_3 + a_4 \\ 2a_3 + 2a_5 - 2a_1 - a_4 \end{bmatrix}$.

(a) **(2 points)** Find the standard matrix A representing L.

$A =$ ☐

(b) **(2 points)** Find $L\left(\begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \\ 1 \end{bmatrix}\right)$.

☐

5.(4 points) Find a, b, c such that the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ a & b & c \\ 0 & 1 & -1 \end{bmatrix}$ has eigenvalue 0,1,2.

$a =$ ☐ $b =$ ☐ $c =$ ☐

6.(4 points) E is a 4×4 matrix of the form $E = \begin{bmatrix} 1 & 1 & 1 & 1 \\ y & x & z & w \\ 2 & 3 & 2 & 3 \\ a & b & c & d \end{bmatrix}$, and $\det(E) = 5$.

Let $F = \begin{bmatrix} -5a & -5b & -5c & -5d \\ 2y & 2x & 2z & 2w \\ 2-3y & 3-3x & 2-3z & 3-3w \\ 1 & 1 & 1 & 1 \end{bmatrix}$. Compute $\det(F)$.

$\det(F) =$ ☐

7.(3 points) Let A and B be 3×3 matrices with $\det(A) = 5$ and $\det(B) = 2$. Set

$$C = B^3 \cdot A^{-1} \cdot (A^T)^4 \cdot (3B) \cdot (B^T)^{-1}. \quad \text{Compute } \det(C).$$

$\det(C) =$ ☐

8.(3 points) Given a matrix $A = \begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$, we know that there are a non-singular matrix P and a diagonal matrix D such that $P^{-1}AP = D$. Find such a matrix D.

$D =$ ☐

9.(6 points) Let $A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$.

(a) **(4 points)** Find $\operatorname{adj} A$.

$\operatorname{adj} A =$ ☐

(b) **(2 points)** Find A^{-1}.

$A^{-1} =$ ☐

10.(6 points) The plane W in space $\mathbb{R}^4$ is defined by the equations $x + 2y + z + 2w = 0, 2x + y + 2z + w = 0$, or we can view W as a subspace of $\mathbb{R}^4$,

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}, x + 2y + z + 2w = 0, 2x + y + 2z + w = 0 \right\}.$$

(a) **(3 points)** Find an orthogonal basis for W.

☐

(b) **(3 points)** Find the distance d between $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ and W.

$d =$ ☐

SECTION II: MULTIPLE CHOICE–NO PARTIAL CREDIT

For Problems 11 through 15, circle **only one** (the correct) answer for each part. **No Partial credit.**

11.(6 points) Determine whether the following are true or false:

(a) **(1 points)** If A, B are $n \times n$ matrices, then $\det(A+B) = \det(A) + \det(B)$. True False

(b) **(1 points)** All eigenvalues of a real symmetry matrix are real and distinct. True False

(c) **(1 points)** Real symmetry matrices are all diagonalizable. True False

(d) **(1 points)** Every inner product space has an orthogonal basis. True False

(e) **(1 points)** Two similar matrices share the same eigenvalues. True False

(f) **(1 points)** If A is a real orthogonal matrix, then then $\big(\det(A)\big)^2 = 1$ True False

12.(4 points) If A is a 3×3 matrix with eigenvalues 0,3,-2 whose associated eigenvectors are

$$\begin{bmatrix}1\\0\\1\end{bmatrix}, \quad \begin{bmatrix}0\\1\\2\end{bmatrix}, \quad \begin{bmatrix}2\\2\\0\end{bmatrix},$$

respectively, then A^3 has an eigenvalue λ whose associated eigenvector is $\mathbf{v}$.

A. $\lambda = 9, \ \mathbf{v} = \begin{bmatrix}1\\0\\1\end{bmatrix}$.

B. $\lambda = 0, \ \mathbf{v} = \begin{bmatrix}0\\1\\2\end{bmatrix}$.

C. $\lambda = 27, \ \mathbf{v} = \begin{bmatrix}1\\0\\1\end{bmatrix}$.

D. $\lambda = -2, \ \mathbf{v} = \begin{bmatrix}1\\1\\0\end{bmatrix}$.

E. $\lambda = -8, \ \mathbf{v} = \begin{bmatrix}1\\1\\0\end{bmatrix}$.

13.(4 points) If A is a 4×4 matrix and $\det(A) = 3$, then what is $\det\big((\mathrm{adj}\ A)^{-1}\big)$?

A. $\frac{1}{27}$.

B. 3.

C. $\frac{1}{3}$.

D. 1.

E. 27.

14.(6 points) Determine if each of the following matrix is diagonalizable.

(a) **(2 points)** $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 3 & 4 \\ 5 & 4 & 3 \end{bmatrix}$. True False

(b) **(2 points)** $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$. True False

(c) **(2 points)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. True False

15.(4 points) Let W be a subspace of $\mathbb{R}^3$ with an orthogonal basis $\left\{ \mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$.

We can write $\mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$ as $\mathbf{u} = \mathbf{u_1} + \mathbf{u_2}$, where $\mathbf{u_1} \in W, \ \mathbf{u_2} \in W^{\perp}$. What are $\mathbf{u_1}, \mathbf{u_2}$?

A. $\mathbf{u_1} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \ \mathbf{u_2} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$.

B. $\mathbf{u_1} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}, \ \mathbf{u_2} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$.

C. $\mathbf{u_1} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \ \mathbf{u_2} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$.

D. $\mathbf{u_1} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}, \ \mathbf{u_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

E. $\mathbf{u_1} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{u_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

SECTION III: MULTI-STEP PROBLEMS – PARTIAL CREDIT

Show all work (**no work - no credit!**) and display computing steps. Write clearly.

16.(10 points) Given four vectors $\mathbf{u_1} = \begin{bmatrix} 1 \\ -3 \\ 5 \\ 1 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} 5 \\ -5 \\ 9 \\ 7 \end{bmatrix}$, $\mathbf{u_3} = \begin{bmatrix} 7 \\ -1 \\ 3 \\ 11 \end{bmatrix}$, $\mathbf{u_4} = \begin{bmatrix} 4 \\ -2 \\ 4 \\ 6 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ 12 \\ 22 \end{bmatrix}$, we set

$$W = \text{span}\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}, \ A = \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \mathbf{u_3} \end{bmatrix}$$

(a) **(3 points)** Find an orthonormal basis for W.

(b) **(3 points)** Find an orthonormal basis for $W^{\perp}$.

(c) **(2 points)** Find $\mathbf{u_1} = \text{Proj}_W \mathbf{b} \in W, \mathbf{u_2} = \text{Proj}_{W^\perp} \mathbf{b} \in W^{\perp}$.

$\mathbf{u_1} =$ $\mathbf{u_2} =$

(d) **(2 points)** Find the least square solution $\hat{\mathbf{x}}$ for the system $A\mathbf{x} = \mathbf{b}$.

$\hat{\mathbf{x}} =$

17.(8 points)

(a) **(4 points)** Given three vectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} -2 \\ 6 \\ -5 \end{bmatrix}$, let $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ and $L\colon \mathbb{R}^3 \to \mathbb{R}^5$ be a linear transformation defined by

$$L(\mathbf{v_1}) = \begin{bmatrix} 5 \\ 5 \\ -2 \\ 8 \\ -2 \end{bmatrix}, L(\mathbf{v_2}) = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 4 \\ -1 \end{bmatrix}, L(\mathbf{v_3}) = \begin{bmatrix} -12 \\ -11 \\ 2 \\ -19 \\ 3 \end{bmatrix}.$$

(i) **(1 points)** Use the determinant to check whether S is a basis for $\mathbb{R}^3$. Justify your answer.

(ii) **(3 points)** Let $\mathbf{v} = \begin{bmatrix} -3 \\ 7 \\ -5 \end{bmatrix}$. Find $L(\mathbf{v})$.

$L(\mathbf{v}) =$ [ ]

(b) **(4 points)** Given three vectors $\mathbf{v'_1} = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}$, $\mathbf{v'_2} = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}$, $\mathbf{v'_3} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, let $S' = \{\mathbf{v'_1}, \mathbf{v'_2}, \mathbf{v'_3}\}$ and $L'\colon \mathbb{R}_3 \to \mathbb{R}_4$ be a linear transformation defined by

$$L'(\mathbf{v'_1}) = \begin{bmatrix} 0 & -1 & 7 & 3 \end{bmatrix}, L'(\mathbf{v'_2}) = \begin{bmatrix} -6 & -1 & 2 & 3 \end{bmatrix}, L'(\mathbf{v'_3}) = \begin{bmatrix} -1 & -1 & 4 & 2 \end{bmatrix}.$$

(i) **(1 points)** Use the determinant to check whether S is a basis for $\mathbb{R}_3$. Justify your answer.

(ii) **(3 points)** Let $\mathbf{v'} = \begin{bmatrix} 0 & -3 & -4 \end{bmatrix}$. Find $L'(\mathbf{v'})$.

$L'(\mathbf{v'}) =$ [ ]

18.(6 points)

Given a system

$$\begin{cases} 2x + 3y - z = & 1 \\ 3x + 5y + 2z = & 8 \\ x - 2y - 3z = & -1 \end{cases},$$

solve the system by the Cramer's rule.

$x =$ ☐ $y =$ ☐ $z =$ ☐

19.(8 points) Let

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

(a) **(3 points)** Find all eigenvalues of A.

(b) **(3 points)** Find three linearly independent eigenvectors of A.

(c) **(2 points)** Find an orthogonal matrix P and a diagonal matrix D such that $P^T AP = D$.

$P =$ $D =$