Math 265 Quiz#2 : 1.5, 1.6

For **Division 7, Section 3**:
For the following linear system

$$\begin{cases} x_1 + x_2 - 2x_3 &= 3 \\ x_2 + x_3 &= 5 \\ 3x_2 + ax_3 &= 5 \end{cases}$$

1. **4 points.** Give a value for a such that the linear system has exactly one solution.
2. **4 points.** Give a value for a such that the linear system has no solution.
2. **2 points.** Does it exist a value for a such that the system has infinitely many solution?

SOLUTION.

1. **4 points.** The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 1 & 1 & 5 \\ 0 & 3 & a & 5 \end{array}\right].$$

   It is row equivalent to the following matrix in row echelon form

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & a-3 & -10 \end{array}\right].$$

   If $a \neq 3$, we can solve x_3, then x_2, x_1 by Gaussian elimination. In that case, we have a unique solution. Therefore, the answer is value of a except 3.
2. **4 points.** If $a = 3$, then the third equation become

$$0 = -10.$$

   Therefore no solution.
2. **2 points.** We have discussed all possibility of a. No case has infinitely many solution. Therefore, no such a value.

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For **Division 8, Section 2**:
For the following linear system

$$\begin{cases} x_1 + x_2 - 2x_3 & = 3 \\ x_2 + x_3 & = 5 \\ 3x_2 + ax_3 & = 15 \end{cases}$$

1. **4 points.** Give a value for a such that the linear system has exactly one solution.
2. **4 points.** Give a value for a such that the linear system has infinitely many solution.
3. **2 points.** Does it exist a value for a such that the system has no solution?

SOLUTION.

1. **4 points.** The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 1 & 1 & 5 \\ 0 & 3 & a & 15 \end{array}\right].$$

   It is row equivalent to the following matrix in row echelon form

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 3 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & a-3 & 0 \end{array}\right].$$

   If $a \neq 3$, we can solve $x_3 = 0$, then x_2, x_1 by Gaussian elimination. In that case, we have a unique solution. Therefore, the answer is value of a except 3.
2. **4 points.** If $a = 3$, then the third equation becomes

$$0 = 0.$$

   So this equation is useless. Then we still can solve this system by Gaussian elimination.

$$\begin{cases} x_1 = & 3r - 2 \\ x_2 = & 5 - r \\ x_3 = & r \\ r & \text{arb.} \end{cases}$$

   Therefore there is infinitely many solutions.
3. **2 points.** We have discussed all possibility of a. No case of a has no solution. Therefore, no such a value.

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