Math 265 Quiz#4: 2.4,2.5

For **Division 7, Section 3**:
Let $V = \mathbb{R}^4$ and W be a subspace of V generated by

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 1 \end{bmatrix}, \ \mathbf{v_2} = \begin{bmatrix} -2 \\ -8 \\ 4 \\ -2 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} -1 \\ -3 \\ 6 \\ 1 \end{bmatrix}, \ \mathbf{v_4} = \begin{bmatrix} -1 \\ -2 \\ 10 \\ 3 \end{bmatrix}, \ \mathbf{v_5} = \begin{bmatrix} 1 \\ 5 \\ 2 \\ 3 \end{bmatrix}.$$

1. **5 points.** Find a basis for W.
2. **5 points.** Find the dimension of W.

SOLUTION.

1. **5 points.**

$$\begin{bmatrix} 1 & -2 & -1 & -1 & 1 \\ 4 & -8 & -3 & -2 & 5 \\ -2 & 4 & 6 & 10 & 2 \\ 1 & -2 & 1 & 3 & 3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & -2 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 8 & 4 \\ 0 & 0 & 2 & 4 & 4 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & -2 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   The leading ones are the first column and the third column. Therefore, we can choose

$$\{\mathbf{v_1}, \mathbf{v_3}\}$$

   as a basis. Of course, there are infinitely many bases of W. For instance, $\{\mathbf{v_2}, \mathbf{v_3}\}$, $\{\mathbf{v_1}, \mathbf{v_4}\}$, $\{\mathbf{v_4}, \mathbf{v_5}\}$, ..., etc.
2. **5 points.** Since the number of elements of a basis is 2. The dimension of W is 2.

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For **Division 8, Section 2**:
Let $V = \mathbb{R}^4$ and W be a subspace of V generated by

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 1 \end{bmatrix}, \ \mathbf{v_2} = \begin{bmatrix} -2 \\ -8 \\ 4 \\ -2 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} -1 \\ -3 \\ 6 \\ 1 \end{bmatrix}, \ \mathbf{v_4} = \begin{bmatrix} -1 \\ -2 \\ 10 \\ 3 \end{bmatrix}, \ \mathbf{v_5} = \begin{bmatrix} 1 \\ 5 \\ 3 \\ 3 \end{bmatrix}.$$

1. **5 points.** Find a basis for W.
2. **5 points.** Find the dimension of W.

SOLUTION.

1. **5 points.**

$$\begin{bmatrix} 1 & -2 & -1 & -1 & 1 \\ 4 & -8 & -3 & -2 & 5 \\ -2 & 4 & 6 & 10 & 3 \\ 1 & -2 & 1 & 3 & 3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & -2 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 4 & 8 & 5 \\ 0 & 0 & 2 & 4 & 2 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & -2 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The leading ones are the first column, the third column and the fifth column. Therefore, we can choose

$$\{\mathbf{v_1}, \mathbf{v_3}, \mathbf{v_5}\}$$

as a basis. Of course, there are infinitely many bases of W. For instance, $\{\mathbf{v_2}, \mathbf{v_3}, \mathbf{v_5}\}$, $\{\mathbf{v_1}, \mathbf{v_4}, \mathbf{v_5}\}$, $\{\mathbf{v_2}, \mathbf{v_4}, \mathbf{v_5}\}$, ..., etc.

2. **5 points.** Since the number of elements of a basis is 3. The dimension of W is 3.

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