Math 265 Quiz#5: 2.6, 2.7, 2.8

For **Division 7, Section 3**:

1. **6 points.** Given a vector $\mathbf{v}$ and an ordered basis $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, where

$$\mathbf{v} = \begin{bmatrix} -2 \\ -1 \\ -6 \end{bmatrix}, \quad \mathbf{v_1} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} -1 \\ 3 \\ -6 \end{bmatrix}.$$

   Find $[\mathbf{v}]_S$.

2. **4 points.** Let

$$A = \begin{bmatrix} 1 & 3 & 7 & -1 \\ -1 & -2 & -5 & 3 \\ 1 & 4 & 9 & 1 \end{bmatrix}.$$

   What the nullity of A? (**2 points**) What is the sum of the nullity of A and the row rank of A? (**2 points**).

Solution.

1. **points.**

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 3 & 7 & 3 & -1 \\ 1 & 1 & -6 & -6 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 6 & 5 \\ 0 & -1 & -5 & -4 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 6 & 5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \Longrightarrow [\mathbf{v}]_S = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

2. **points.**

$$\begin{bmatrix} 1 & 3 & 7 & -1 \\ -1 & -2 & -5 & 3 \\ 1 & 4 & 9 & 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 3 & 7 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 3 & 7 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow \text{nullity } = 2.$$

   The sum of nullity and (row) rank is always equal to the number of columns. Therefore, it is equal to 4.

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For **Division 8, Section 2**:

1. **6 points.** Given a vector $\mathbf{v}$ and an ordered basis $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, where

$$\mathbf{v} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}, \quad \mathbf{v_1} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Find $\left[\mathbf{v}\right]_S$.

2. **4 points.** Let

$$A = \begin{bmatrix} 1 & 3 & 7 & -1 & 2 \\ -1 & -2 & -5 & 3 & -4 \\ 1 & 4 & 9 & 1 & 0 \end{bmatrix}.$$

What the nullity of A? (**2 points**) What is the sum of the nullity of A and the row rank of A? (**2 points**).

SOLUTION.

1. **points.**

$$\begin{bmatrix} 1 & 2 & 1 & \vdots & 2 \\ 3 & 7 & 2 & \vdots & 6 \\ 1 & 1 & 3 & \vdots & 3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 2 & 1 & \vdots & 2 \\ 0 & 1 & -1 & \vdots & 0 \\ 0 & -1 & 2 & \vdots & 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 2 & 1 & \vdots & 2 \\ 0 & 1 & -1 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} 1 & 2 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix} \Longrightarrow \left[\mathbf{v}\right]_S = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$

2. **points.**

$$\begin{bmatrix} 1 & 3 & 7 & -1 & 2 \\ -1 & -2 & -5 & 3 & -4 \\ 1 & 4 & 9 & 1 & 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 3 & 7 & -1 & 2 \\ 0 & 1 & 2 & 2 & -2 \\ 0 & 1 & 2 & 2 & -2 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 3 & 7 & -1 & 2 \\ 0 & 1 & 2 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow \text{nullity} \ = 3.$$

The sum of nullity and (row) rank is always equal to the number of columns. Therefore, it is equal to 5.

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