

Math 265 Quiz#8: 4.1

For Division 7, Section 3:

1. **5 points.** Consider the following transformation $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$,

$$L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} a_1 + a_2 + a_3 \\ a_2 - a_1 \end{bmatrix}.$$

Find the matrix representative A such that $L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = A \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$.

2. **5 points.** Consider the following transformation $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$$L\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \quad L\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix}.$$

(i) **3 points.** Find the matrix representative A such that $L\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = A \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$.

(ii) **2 points.** Find $L\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right)$.

SOLUTION.

1. **5 points.**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}.$$

2. **5 points.**

(i) **3 points.**

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \\ \cdots & \cdots \\ 1 & 2 \\ 3 & 7 \\ 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ \cdots & \cdots \\ 1 & 0 \\ 3 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \cdots & \cdots \\ 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

(ii) **2 points.**

$$L\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = A \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}.$$

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For **Division 8, Section 2**:

1. **5 points.** Consider the following transformation $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$,

$$L \left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \right) = \begin{bmatrix} 2a_1 - a_2 + a_3 \\ a_3 - a_1 \end{bmatrix}.$$

Find the matrix representative A such that $L \left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \right) = A \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$.

2. **5 points.** Consider the following transformation $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$$L \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad L \left(\begin{bmatrix} 2 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}.$$

(i) **3 points.** Find the matrix representative A such that $L \left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \right) = A \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$.

(ii) **2 points.** Find $L \left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} \right)$.

SOLUTION.

1. **5 points.**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}.$$

2. **5 points.**

(i) **3 points.**

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ \dots & \dots \\ 2 & 2 \\ 1 & 0 \\ -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ \dots & \dots \\ 2 & -2 \\ 1 & -2 \\ -1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ \dots & \dots \\ -2 & -2 \\ -3 & -2 \\ 7 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \dots & \dots \\ -2 & 2 \\ -3 & 2 \\ 7 & -4 \end{bmatrix} \Rightarrow A = \begin{bmatrix} -2 & 2 \\ -3 & 2 \\ 7 & -4 \end{bmatrix}.$$

(ii) **2 points.**

$$L \left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} \right) = A \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -3 & 2 \\ 7 & -4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 6 \end{bmatrix}.$$

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