Math 265 Quiz#13: 7.1,7.2

For **Division 7, Section 3**:

1. **6 points.** Find the general solution for

$$\mathbf{x}'(\mathbf{t}) = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix} \mathbf{x}(\mathbf{t}), \quad \mathbf{x}(\mathbf{t}) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

2. **4 points.** For each of the following dynamical systems, determine the nature of the equilibrium point at the origin and describe the phase portrait.

   (i) **2 points.**

$$\mathbf{x}'(\mathbf{t}) = \begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix} \mathbf{x}(\mathbf{t}).$$

   (ii) **2 points.**

$$\mathbf{x}'(\mathbf{t}) = \begin{bmatrix} 2 & -5 \\ -1 & -2 \end{bmatrix} \mathbf{x}(\mathbf{t}).$$

SOLUTION.

1. **6 points.** The characteristic polynomial is $(\lambda - 3)(\lambda + 3) + 8 = \lambda^2 - 1$ and the roots are ± 1. The associated eigenvector are

$$1 \rightsquigarrow \begin{bmatrix} 2 \\ -1 \end{bmatrix}, -1 \rightsquigarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Longrightarrow \mathbf{x}(t) = b_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^t + b_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}.$$

2. **4 points.**

   (i) **2 points.** The characteristic polynomial is $(\lambda - 2)^2 + 5$ and the roots are $2 \pm \sqrt{5}i$. We have complex roots. Hence, the origin is spiral. The real part is $2 > 0$; therefore, it is unstable and goes outward.

   (ii) **2 points.** The characteristic polynomial is $(\lambda - 2)(\lambda + 2) - 5$ and the roots are ± 3. We have real roots with different signs. Hence, the origin is a saddle point.

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For **Division 8, Section 2**:

1. **6 points.** Find the general solution for

$$\mathbf{x}'(\mathbf{t}) = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \mathbf{x}(\mathbf{t}), \quad \mathbf{x}(\mathbf{t}) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

2. **4 points.** For each of the following dynamical systems, determine the nature of the equilibrium point at the origin and describe the phase portrait.

   (i) **2 points.**

$$\mathbf{x}'(\mathbf{t}) = \begin{bmatrix} -3 & 4 \\ 4 & -3 \end{bmatrix} \mathbf{x}(\mathbf{t}).$$

   (ii) **2 points.**

$$\mathbf{x}'(\mathbf{t}) = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix} \mathbf{x}(\mathbf{t}).$$

SOLUTION.

1. **6 points.** The characteristic polynomial is $(\lambda - 2)(\lambda + 2) + 3 = \lambda^2 - 1$ and the roots are ± 1. The associated eigenvector are

$$1 \rightsquigarrow \begin{bmatrix} 3 \\ -1 \end{bmatrix}, -1 \rightsquigarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Longrightarrow \mathbf{x}(t) = b_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^t + b_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}.$$

2. **4 points.**

   (i) **2 points.** The characteristic polynomial is $(\lambda + 3)^2 - 16$ and the roots are $-7, 1$. We have real roots with different signs. Hence, the origin is a saddle point.

   (ii) **2 points.** The characteristic polynomial is $(\lambda + 3)^2 + 16$ and the roots are $-3 \pm 4i$. We have complex roots. Hence, the origin is spiral. The real part is $-3 < 0$; therefore, it is stable and goes inward.

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