Worked Out Homework 2
MA 303 Fall 2011 (Aaron N. K. Yip)
Friday, Sept. 16, in class

1. Find the general solution of $\frac{dX}{dt} = BX$, $\frac{dX}{dt} = CX$ and $\frac{dX}{dt} = DX$ where B, C, D are the following matrices:

$$B = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

Note 1: The above matrices are the same as the Hand-Written Hw 1, #2. To solve for the system with matrix B, please refer to textbook, p. 429, #17.

Note 2: The problem with matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ from the Hand-Written Hw 1, #2 is somewhat complicated, but doable – see p. 430, #18 for your curiosity and amusement.

2. (Exponential matrices)

   (a) Find e^{At} for $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Your answer should not contain any complex numbers.
   (Hint: make use of the general solution formula.)

   (b) Find e^{Bt} for $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
   (Hint: make use of the general solution formula.)

   (c) Find $e^{(A+B)t}$.
   (Hint: make use of the general solution formula.)

   (d) Is $e^{(A+B)t} = e^{At}e^{Bt} = e^{Bt}e^{At}$?

3. (For this problem you can make use of matlab for assistance.)

   (a) Consider the multiple spring-mass system as described in Textbook p. 356 and p. 406. For the parameters stated in p. 406, Example 3, find the *explicit analytical form* of the solution when $x_1(0) = 1$, $\dot{x}_1(0) = 0$, $x_2(0) = -0.5$, $\dot{x}_2(0) = 0$.
   Plot the graphs of $x_1(t)$ and $x_2(t)$ versus $t \geq 0$ in the SAME FIGURE. Set the range of t to be $0 \leq t \leq 30$. Make sure you label the curves and use different styles to distinguish the two curves.

(b) Consider the same multiple spring-mass system as before with the same masses and spring constants as before. But now let the *frictional coeffcients* between the masses and the table be $\mu_1 = 1$ and $\mu_2 = 0.5$. i.e the frictional force between the mass and the table is given by: $-\mu_1 \dot{x}_1(t)$ and $-\mu_2 \dot{x}_2(t)$.

Find the *explicit analytical form* of the solution when $x_1(0) = 1$, $\dot{x}_1(0) = 0$, $x_2(0) = -0.5$, $\dot{x}_2(0) = 0$.

Plot the graphs of $x_1(t)$ and $x_2(t)$ versus $t \geq 0$ in the SAME FIGURE. Make sure you label the curves and use different styles to distinguish the two curves.

Note 1: you might need to *adjust the range of* t in the graph in order to reveal the most "relevant" information about the behavior of the solutions.

Note 2: you *should* follow the steps outlined below in order to demonstrate your thought process:

i. Write down the system of differential equations

ii. Find the *general solution*

iii. Find the *exact analytical solution* with the *given initial data.*

iv. Plot the graphs of $x_1(t)$ and $x_2(t)$ versus t.