Worked Out Homework 2

MA 303 Fall 2011 (Aaron N. K. Yip)

Friday, Sept. 16, in class

1. Find the general solution of $\frac{dX}{dt} = BX$, $\frac{dX}{dt} = CX$ and $\frac{dX}{dt} = DX$ where B, C, D are the following matrices:

$$B = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

Note 1: The above matrices are the same as the Hand-Written Hw 1, #2. To solve for the system with matrix B, please refer to textbook, p. 429, #17.

Note 2: The problem with matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ from the Hand-Written Hw 1, #2 is somewhat complicated, but doable – see p. 430, #18 for your curiosity and amusement.

2. (Exponential matrices)

   (a) Find e^{At} for $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Your answer should not contain any complex numbers.
   (Hint: make use of the general solution formula.)

   (b) Find e^{Bt} for $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
   (Hint: make use of the general solution formula.)

   (c) Find $e^{(A+B)t}$.
   (Hint: make use of the general solution formula.)

   (d) Is $e^{(A+B)t} = e^{At}e^{Bt} = e^{Bt}e^{At}$?

3. (For this problem you can make use of matlab for assistance.)

   (a) Consider the multiple spring-mass system as described in Textbook p. 356 and p. 406. For the parameters stated in p. 406, Example 3, find the *explicit analytical form* of the solution when $x_1(0) = 1$, $\dot{x}_1(0) = 0$, $x_2(0) = -0.5$, $\dot{x}_2(0) = 0$.

   Plot the graphs of $x_1(t)$ and $x_2(t)$ versus $t \geq 0$ in the SAME FIGURE. Set the range of t to be $0 \leq t \leq 30$. Make sure you label the curves and use different styles to distinguish the two curves.

MA303 Fall 2011 (Yip) HW2 Solution

#1 B: $\lambda = 3, 3, 3$

Only $\underline{\underline{1}}$ eigenvector (refer to HW1 #2)

$$V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

(Refer to Textbook Pg 429 #17)

$$X_1(t) = c_1 e^{3t} V_1 = c_1 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$X_2(t) = c_2 e^{3t} \left[t V_1 + W \right]$$

where $(A - 3I) W = V_1$

$$\Rightarrow W = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

i.e. $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow$

$\beta = 1$

$\gamma = 0$

$\alpha = \text{free} \overset{\text{set to}}{=} 0$

$$X_3(t) = C_3 e^{3t}\left[\frac{t^2}{2} V_1 + tW + U \right]$$

where $(A - 3I)\,U = W$

i.e.

$$\left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\beta = 0$

$\gamma = 1$

α - free (set to $= 0$)

$$\Rightarrow \quad U = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Hence general solution

$$\begin{aligned} X(t) = {} & c_1 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ & + c_2 e^{3t}\left[t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \\ & + c_3 e^{3t}\left[\frac{t^2}{2}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] \end{aligned}$$

$$C = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad \lambda = 3, 3, 2$$

$\lambda = 3$, Two eigenvectors: $V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda = 2$, one eigenvector $V_3 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

General solution:

$$\boxed{X(t) = c_1 e^{3t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{2t} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}}$$

$$D = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{bmatrix} \qquad \lambda = 3, 3, 2$$

$\lambda = 3$, ONE eigenvector $\quad V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 2$, one eigenvector $\quad V_2 = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$

General solution is

$$\boxed{X(t) = c_1 e^{3t}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2 e^{3t}\left[t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \vec{W} \right] + C_2 e^{2t}\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}}$$

$$\vec{W} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

where $(\cancel{A}D - 3I)W = V_1$

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} \beta + 2\gamma = 1 \\ \gamma = 0 \\ \alpha = \text{free} (= 0) \end{array} \Rightarrow \beta = 1$$

#2 a) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ find e^{At}

~~[illegible]~~ find (λ, V)

$\det(A - \lambda I) = \det\begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 + 1 = 0 \quad \lambda = \pm i$

$\lambda = i, \quad (A - iI)V = 0 \Rightarrow \left(\begin{array}{cc|c} -i & -1 & 0 \\ 1 & -i & 0 \end{array}\right)$

$\rightarrow \left(\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right)$

$V_1 = \begin{pmatrix} i \\ 1 \end{pmatrix}$

$\lambda = -i, \quad V_2 = \begin{pmatrix} -i \\ 1 \end{pmatrix}$

$e^{At} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{it} & \\ & e^{-it} \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1}$

Method 1

$$e^{At} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{it} & \\ & e^{-it} \end{bmatrix} \frac{\begin{bmatrix} 1 & i \\ -1 & i \end{bmatrix}}{2i}$$

$$= \begin{bmatrix} ie^{it} & -ie^{-it} \\ e^{it} & e^{-it} \end{bmatrix} \begin{bmatrix} 1 & i \\ -1 & i \end{bmatrix} \Big/ 2i$$

$$= \begin{bmatrix} i\left(e^{it}+e^{-it}\right) & -e^{it}+e^{-it} \\ e^{it}-e^{-it} & i\left(e^{it}+e^{-it}\right) \end{bmatrix} \Big/ 2i$$

$$= \begin{bmatrix} \frac{e^{it}+e^{-it}}{2} & -\frac{e^{it}-e^{-it}}{2i} \\ \frac{e^{it}-e^{-it}}{2i} & \frac{e^{it}+e^{-it}}{2} \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}}$$

(Recall: $e^{it} = \cos t + i\sin t$, $e^{-it} = \cos t - i\sin t$

$\dfrac{e^{it} + e^{-it}}{2} = \cos t$, $\dfrac{e^{it} - e^{-it}}{2i} = \sin t$)

<u>Method 2</u> Make use of general solution

$\lambda_1 = i$; $V_1 = \begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$X_1(t) = \cancel{e^{0t}} \left[\cos t \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \sin t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix}$$

$$X_2(t) = \cancel{e^{0t}} \left[\sin t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \cos t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} -\sin t & \cos t \\ \cos t & \sin t \end{bmatrix}$$

$$e^{At} = [\Phi(t)]\,[\Phi(0)]^{-1}$$

$$= \begin{bmatrix} -S & C \\ C & S \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -S & C \\ C & S \end{bmatrix} \begin{bmatrix} 0 & +1 \\ +1 & 0 \end{bmatrix} \Big/ +1$$

$$= \boxed{\begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}}$$

$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\lambda = 0, 0$

$(B - \lambda I) V_1 = 0 \Rightarrow \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \rightarrow \beta = 0$

α (free) $= 1$

$V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ONLY ONE Eigenvector

General solution: $X_1(t) = e^{0t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$X_2(t) = e^{0t}\left[t\begin{pmatrix} 1 \\ 0 \end{pmatrix} + W\right]$

where W solves: $(A - \lambda I) W = V_1$

$\left(\begin{array}{cc|c} 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$ $\beta = 1$

$\alpha = 0$ (free)

Hence $X_1(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $X_2(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$

$$\Phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

$$e^{Bt} = [\Phi(t)]\,[\Phi(0)]^{-1} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \cancel{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1}}$$

$$= \boxed{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}$$

$$C = A + B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad \lambda = 0, 0$$

$$(C - 0I)V = 0 \Rightarrow \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} \alpha = 0 \\ \beta = 1 \text{ (free)} \end{array}$$

ONLY ONE Eigenvector

General solution

$$X_1(t) = e^{0t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad X_2(t) = e^{0t}\left[t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + W\right]$$

where $(C - 0I)W = V_1$

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{l} \alpha = 1 \\ \beta = 0 \text{ (free)} \end{array} \Rightarrow W = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Hence $X_1(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $X_2(t) = \begin{bmatrix} 1 \\ t \end{bmatrix}$

$$\Phi(t) = \begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}$$

$$e^{Ct} = \left[\Phi(t)\right]\left[\Phi(0)\right]^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \Big/ -1$$

$$= \boxed{\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}}$$

<u>Check</u> $e^{Ct} = e^{At} e^{Bt} = e^{Bt} e^{At}$?

$$\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \;?$$

$$\neq \begin{bmatrix} c & tc-1 \\ s & ts+c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$$

$$\neq \begin{bmatrix} c+ts & -s+tc \\ s & c \end{bmatrix}$$

No, No, No !!!

#3 (a) Already solved by textbook, Pg 407

$$\vec{y} = c_1 \begin{bmatrix} 3\cos t \\ 2\cos t \\ -3\sin t \\ -2\sin t \end{bmatrix} + c_2 \begin{bmatrix} 3\sin t \\ 2\sin t \\ 3\cos t \\ 2\cos t \end{bmatrix}$$

$$+ c_3 \begin{bmatrix} 3\cos 2t \\ -4\cos 2t \\ -6\sin 2t \\ 8\sin 2t \end{bmatrix} + c_4 \begin{bmatrix} 3\sin 2t \\ -4\sin 2t \\ 6\cos 2t \\ -8\cos 2t \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$$

$(y_1 = x_1,\ y_2 = x_2,\ y_3 = \dot{x}_1,\ y_4 = \dot{x}_2)$

$\boxed{\text{at } t=0}$

$$\begin{bmatrix} 1 \\ -0.5 \\ 0 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ -4 \\ 0 \\ 0 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 8 \end{bmatrix}$$

Make use of Matlab (or by hand) :

$$\begin{bmatrix} 3 & 0 & 3 & 0 \\ 2 & 0 & -4 & 0 \\ 0 & 3 & 0 & 6 \\ 0 & 2 & 0 & 8 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 & 0 \\ 2 & 0 & -4 & 0 \\ 0 & 3 & 0 & 6 \\ 0 & 2 & 0 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -0.5 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

(see Matlab script) (1)

$$= \begin{array}{cl} \begin{bmatrix} 0.1389 \\ 0 \\ 0.1944 \\ 0 \end{bmatrix} & \begin{array}{l} \leftarrow c_1 \\ \leftarrow c_2 \\ \leftarrow c_3 \\ \leftarrow c_4 \end{array} \end{array}$$

Hence
$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = 0.1389 \begin{bmatrix} 3\cos t \\ 2\cos t \\ -3\sin t \\ -2\sin t \end{bmatrix} + 0.1944 \begin{bmatrix} 3\cos 2t \\ -4\cos 2t \\ -6\sin 2t \\ 8\sin 2t \end{bmatrix}$$

$$X_1(t) = 0.1389\,(3)\cos t + 0.1944\,(3)\cos 2t$$

$$\boxed{X_1(t) = 0.4169\cos t + 0.5832\cos 2t}$$

$$X_2(t) = 0.1389\,(2)\cos t - 0.1944\,(4)\cos 2t$$

$$\boxed{X_2(t) = 0.2778\cos t - 0.7776\cos 2t}$$

Plot 2

```
% Setting up of matrix M

M=[3 0 3 0
2 0 -4 0
0 3 0 6
0 2 0 8]

M =

     3     0     3     0
     2     0    -4     0
     0     3     0     6
     0     2     0     8

% Setting up of the right-hand-side of MC=B

B=[1
-0.5
0
0
]

B =

    1.0000
   -0.5000
         0
         0

% Solve for C: in MC=B. Solution give by C=M^(-1)B

inv(M)*B

ans =

    0.1389
         0
    0.1944
         0
```

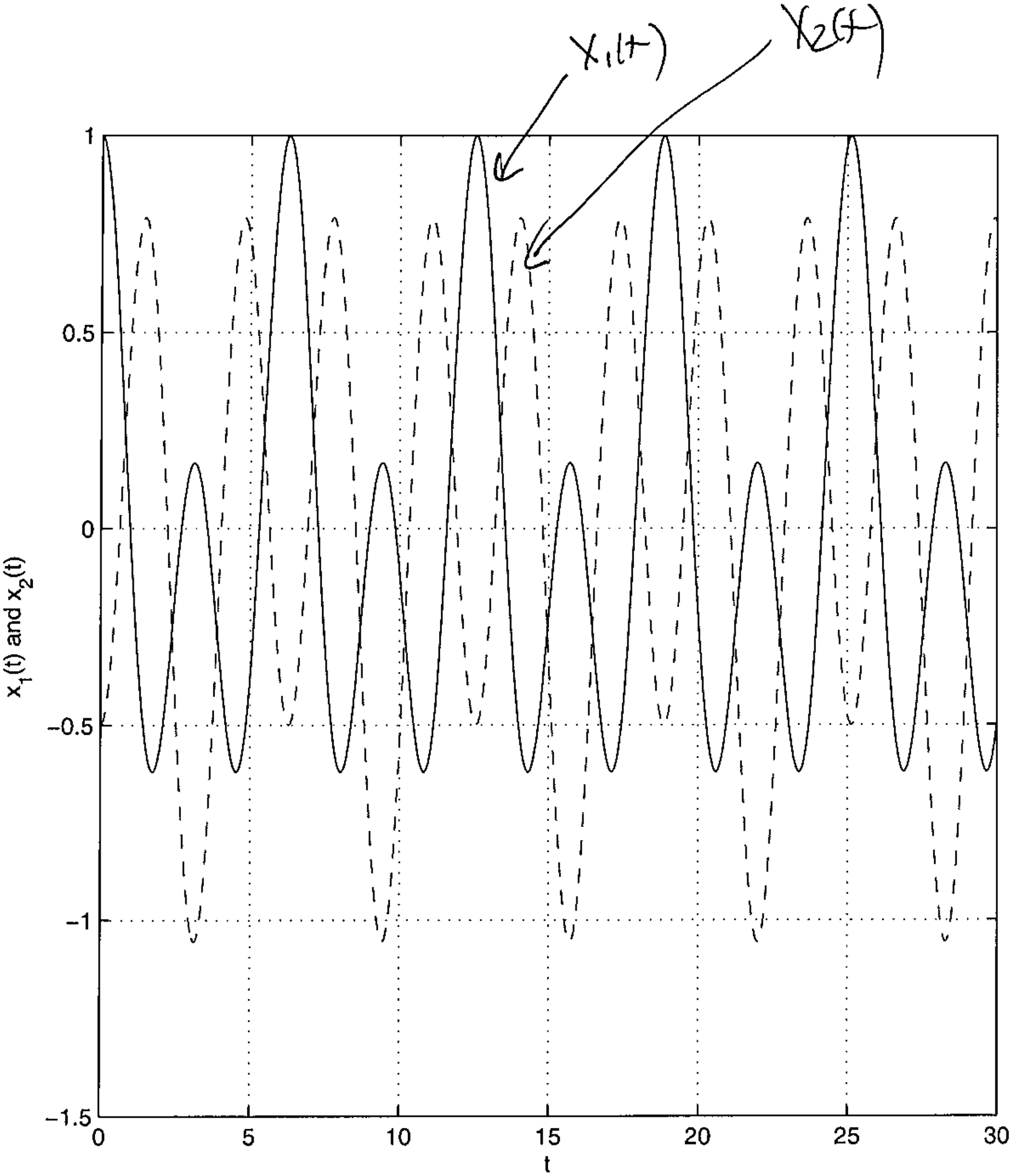
1

```
% Plotting of x1(t) and x2(t)

t=linspace(0,30,1000);
x1=0.4160*cos(t)+0.5832*cos(2*t);
x2=0.2778*cos(t)-0.7776*cos(2*t);
plot(t,x1)
hold on
plot(t,x2,'r--')
grid
xlabel('x_1(t) and x_2(t)')
ylabel('x_1(t) and x_2(t)')
xlabel('t')
```

2

#3(a)



(b) $m_1\ddot{x}_1 = -(k_1+k_2)x_1 + k_2x_2 - \underline{\underline{\mu_1\dot{x}_1}}$

$m_2\ddot{x}_2 = k_2x_1 - (k_2+k_3)x_2 - \underline{\underline{\mu_2\dot{x}_2}}$

new terms

Modified matrix:

$$\dot{\vec{y}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3/2 & -\mu_1/m_1 & 0 \\ 4/3 & -3 & 0 & -\mu_2/m_2 \end{bmatrix} \vec{y}$$

$\frac{0.5(4)}{9}$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3/2 & -1/2 & 0 \\ 4/3 & -2 & 0 & -2/9 \end{bmatrix} \vec{y}$$

$\mu_1 = 1,\ m_1 = 2$

$\mu_2 = 0.5,\ m_2 = 9/4$

matlab 3

$$\lambda_1, \lambda_2 = -0.1793 \pm 1.8327 i$$

$$V_1, V_2 = \underbrace{\begin{bmatrix} -0.0335 \\ 0.0908 \\ 0.6340 \\ -0.5986 \end{bmatrix}}_{u_1} \pm i \underbrace{\begin{bmatrix} -0.3427 \\ 0.3177 \\ 0 \\ 0.1095 \end{bmatrix}}_{w_1}$$

$$\lambda_3, \lambda_4 = -0.1818 \pm 0.7462 i$$

$$V_3, V_4 = \underbrace{\begin{bmatrix} -0.5819 \\ -0.5374 \\ 0.1058 \\ 0.1272 \end{bmatrix}}_{u_2} \pm i \underbrace{\begin{bmatrix} 0 \\ -0.0395 \\ -0.4342 \\ -0.3938 \end{bmatrix}}_{w_2}$$

General Solution.

$$\vec{y}(t) = C_1 e^{-0.1793t}\left[\cos(1.8327t)\,U_1 - \sin(1.8327t)\,W_1\right]$$

$$+ C_2 e^{-0.1793t}\left[\sin(1.8327t)\,U_1 + \cos(1.8327t)\,W_1\right]$$

$$+ C_3 e^{-0.1818t}\left[\cos(0.7462t)\,U_2 - \sin(0.7462t)\,W_2\right]$$

$$+ C_4 e^{-0.1818t}\left[\sin(0.7462t)\,U_2 + \cos(0.7462t)\,W_2\right]$$

Set $t=0 \Longrightarrow$

$$\begin{bmatrix} 1 \\ -0.5 \\ 0 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} \\ u_1 \\ \\ \end{bmatrix} + c_2 \begin{bmatrix} \\ w_1 \\ \\ \end{bmatrix} + c_3 \begin{bmatrix} \\ u_2 \\ \\ \end{bmatrix} + c_4 \begin{bmatrix} \\ w_2 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} u_1 & w_1 & u_2 & w_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} u_1 & w_1 & u_2 & w_2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -0.5 \\ 0 \\ 0 \end{bmatrix}$$

(from Matlab).

$$= \begin{bmatrix} -0.2184 \\ -2.2285 \\ -0.3935 \\ -0.4148 \end{bmatrix} \begin{array}{l} \leftarrow c_1 \\ \leftarrow c_2 \\ \leftarrow c_3 \\ \leftarrow c_4 \end{array}$$

4

Hence

$$
\begin{aligned}
X_1(t) = & -0.2184\, e^{-0.1793t}\left[-0.0335\cos(1.8327t) + 0.3427\sin(1.8327t)\right] \\
& -2.2285\, e^{-0.1793t}\left[-0.0335\sin(1.8327t) - 0.3427\cos(1.8327t)\right] \\
& -0.3935\, e^{-0.1818t}\left[-0.5819\cos(0.7462t)\right] \\
& -0.4148\, e^{-0.1818t}\left[-0.5819\sin(0.7462t)\right]
\end{aligned}
$$

t

$$
\begin{aligned}
X_2(t) = & -0.2184\, e^{-0.1793t}\left[0.0908 \cos(1.8327t) - 0.3177 \sin(1.8327t)\right] \\
& -2.2285\, e^{-0.1793t}\left[0.0908 \sin(1.8327t) + 0.3177 \cos(1.8327t)\right] \\
& -0.3935\, e^{-0.1818t}\left[-0.5374 \cos(0.7462t) + 0.0395 \sin(0.7462t)\right] \\
& -0.4148\, e^{-0.1818t}\left[0.5374 \sin(0.7462t) - 0.0395 \cos(0.7462t)\right]
\end{aligned}
$$

plot 5

```
% Setting up of matrix A

A=[0 0 1 0
0 0 0 1
-2 3/2 -1/2 0
4/3 -2 0 -2/9]

A =

         0         0    1.0000         0
         0         0         0    1.0000
   -2.0000    1.5000   -0.5000         0
    1.3333   -2.0000         0   -0.2222

% Finding eigenvectors and eigenvalues of A

[V D]=eig(A)
```

③

```
V =

  -0.0335 - 0.3427i  -0.0335 + 0.3427i  -0.5819             -0.5819
   0.0908 + 0.3177i   0.0908 - 0.3177i  -0.5374 - 0.0395i  -0.5374 + 0.0395i
   0.6340             0.6340             0.1058 - 0.4342i   0.1058 + 0.4342i
  -0.5986 + 0.1095i  -0.5986 - 0.1095i   0.1272 - 0.3938i   0.1272 + 0.3938i


D =

  -0.1793 + 1.8327i        0                  0                  0
        0            -0.1793 - 1.8327i        0                  0
        0                  0            -0.1818 + 0.7462i        0
        0                  0                  0            -0.1818 - 0.7462i

% Setting up of matrix M

M(1:4, 1:4)=0

M =

     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0

M(:,1)=[-0.0335, 0.0908, 0.6340, -0.5986]

M =

   -0.0335         0         0         0
    0.0908         0         0         0
    0.6340         0         0         0
   -0.5986         0         0         0

M(:,2)=[-0.3427, 0.3177, 0, 0.1095]

M =

   -0.0335   -0.3427         0         0
    0.0908    0.3177         0         0
    0.6340         0         0         0
   -0.5986    0.1095         0         0

M(:,3)=[-0.5819, -0.5374, 0.1058, 0.1272]
```

```
M =

   -0.0335   -0.3427   -0.5819         0
    0.0908    0.3177   -0.5374         0
    0.6340         0    0.1058         0
   -0.5986    0.1095    0.1272         0

M(:,4)=[0, -0.0395, -0.4342, -0.3938]

M =

   -0.0335   -0.3427   -0.5819         0
    0.0908    0.3177   -0.5374   -0.0395
    0.6340         0    0.1058   -0.4342
   -0.5986    0.1095    0.1272   -0.3938

% Setting up of right hand side of MC=B

B=[1, -0.5, 0, 0]

B =

    1.0000   -0.5000         0         0

B=[1
-0.5
0
0]

B =

    1.0000
   -0.5000
         0
         0

% Solving for C in MC=B: C=M^(-1)B

inv(M)*B

ans =

   -0.2184
   -2.2285
   -0.3935
   -0.4148

% run script hw23.m to plot x1(t) and x2(t)
hw23b
```

4

5

5

```
t=linspace(0,30,1000);

x1 = ...
-0.2184*exp(-0.1793*t).*(-0.0335*cos(1.8327*t) + 0.3427*sin(1.8327*t)) ...
-2.2285*exp(-0.1793*t).*(-0.0335*sin(1.8327*t) - 0.3427*cos(1.8327*t)) ...
-0.3935*exp(-0.1818*t).*(-0.5819*cos(0.7462*t)) ...
-0.4148*exp(-0.1818*t).*(-0.5819*sin(0.7462*t));

x2 = ...
-0.2184*exp(-0.1793*t).*(0.0908*cos(1.8327*t) - 0.3177*sin(1.8327*t)) ...
-2.2285*exp(-0.1793*t).*(0.0908*sin(1.8327*t) + 0.3177*cos(1.8327*t)) ...
-0.3935*exp(-0.1818*t).*(-0.5374*cos(0.7462*t) + 0.0395*sin(0.7462*t)) ...
-0.4148*exp(-0.1818*t).*(-0.5374*sin(0.7462*t) - 0.0395*cos(0.7462*t));

figure(2)
plot(t,x1)
hold on
plot(t,x2,'r--')
hold off
grid on
```

<u>m-file</u> hw23b.m

to plot $x_1(t)$, $x_2(t)$

Hw 2

#3(b)