

**Worked Out Homework 4**  
**MA 303 Fall 2011 (Aaron N. K. Yip)**  
**Friday, Oct. 28, in class**

1. Consider the systems from the textbook (Boyce-DiPrima, 9th-ed.) section 9.4, page 530, #1, and #2.

For each problem, do the following:

- (a) Find all the critical points;
- (b) Determine the stability type of all the critical points;
- (c) Use matlab pplane8 (available on Purdue Career Account, or other appropriate softwares) to draw *a few but sufficiently many* trajectories so as to describe the overall, long time behavior of the system.

(Note that both  $x$  and  $y$  are restricted to be non-negative numbers.)

2. Consider the system from the textbook (Boyce-DiPrima, 9th-ed.) section 9.4, page 533, #17.

- (a) Find all the critical points;
- (b) Determine the stability of the critical points;
- (c) Determine the *critical values* of  $\alpha$  such that the *behavior of the system changes*.
- (d) Illustrate your results with phase plots (matlab or other computer softwares).

Note the restrictions: (i)  $\alpha$  is a positive number; (ii)  $x$  and  $y$  are non-negative numbers.

3. Do Textbook Section 9.7, p. 564, #16. Illustrate your results with phase plots (matlab or other computer softwares).

Note: The parameter  $\mu$  can be any number, positive or negative.

4. Find the inverse Laplace Transform of the following function:

$$\frac{1}{s^4 + 4}$$

(Hint: Write  $s^4 + 4$  as  $s^4 + 4s^2 + 4 - 4s^2 = (s^2 + 2)^2 - (2s)^2 = (s^2 + 2 - 2s)(s^2 + 2 + 2s)$  and then apply partial fraction and completing square.)

5. By means of the suggested procedure, find the inverse Laplace Transform of the following functions:

$$\frac{1}{(s^2 + 1)^2} \quad \text{and} \quad \frac{s}{(s^2 + 1)^2}.$$

(Write  $(s^2 + 1)^2$  as  $(s + i)^2(s - i)^2$  and then perform partial fraction using complex numbers. Express your final answer in terms of *real functions*. This can be achieved by using:  $\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos \theta$  and  $\frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin \theta$ .)