

Properties and Formulas for Laplace Transform

Let $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$ and $G(s) = \mathcal{L}\{g(t)\} = \int_0^{\infty} e^{-st} g(t) dt$.

In the following a and b are arbitrary constants and n is some positive integer.

$$\mathcal{L}\{af + bg\} = aF(s) + bG(s) \quad (1)$$

$$\mathcal{L}\{f'\} = sF(s) - f(0) \quad (2)$$

$$\mathcal{L}\{f''\} = s^2F(s) - sf(0) - f'(0) \quad (3)$$

$$\mathcal{L}\{f^{(n)}\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0) \quad (4)$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s - a) \quad (5)$$

$$\mathcal{L}\{f(t - a)u(t - a)\} = e^{-as}F(s) \quad (6)$$

$$\mathcal{L}\{tf(t)\} = -F'(s) \quad (7)$$

$$\mathcal{L}\{f * g(t)\} = F(s)G(s) \quad (8)$$

$$\mathcal{L}\{\delta(t)\} = 1 \quad (9)$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad (10)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s - a} \quad (11)$$

$$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s - a)^{n+1}} \quad (12)$$

$$\mathcal{L}\{\cos bt\} = \frac{s}{s^2 + b^2} \quad (13)$$

$$\mathcal{L}\{\sin bt\} = \frac{b}{s^2 + b^2} \quad (14)$$

$$\mathcal{L}\{e^{at} \cos bt\} = \frac{s - a}{(s - a)^2 + b^2} \quad (15)$$

$$\mathcal{L}\{e^{at} \sin bt\} = \frac{b}{(s - a)^2 + b^2} \quad (16)$$